LOW-DENSITY PARITY-CHECK CODES FOR ARQ-BASED COOPERATIVE DIVERSITY SCHEMES

BY

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This thesis, written by **HUSSAIN ALI** under the direction of his thesis adviser and approved by his thesis committee, has been presented to and accepted by the Dean of Graduate Studies, in partial fulfillment of the requirements for the degree of **MASTER OF SCIENCE IN TELECOMMUNICATION ENGINEERING**.

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Dedicated to my beloved parents

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In the name of Allah, the Most Beneficent, the Most Merciful

All praise be to Allah (The One and The Only Creator of everything) for His limitless blessings. May Allah bestow peace and His choicest blessings on His last prophet, Hazrat Muhammad (Peace Be Upon Him), his family (May Allah be pleased with them), his companions (May Allah be pleased with them) and his followers.

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THESIS ABSTRACT

NAME:	Hussain Ali
TITLE OF STUDY:	Low-Density Parity-Check codes for ARQ-based Cooperative Diversity Schemes
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Diversity techniques are used to mitigate fading and channel impairments of wireless communication channels. Cooperative diversity or user cooperation achieves the diversity gain without adding physical antennas to the users or mobile stations. In this work, cooperative diversity is used as the main framework.

The cooperative diversity has been extended to coded cooperative diversity with the addition of rate-compatible punctured convolutional (RCPC) codes. Low-density parity-check (LDPC) codes are capacity-achieving codes and they have better error correction capabilities than RCPC codes. Therefore, LDPC codes can be integrated in cooperative diversity scheme to increase the diversity gain. Punctured and extended LDPC codes have also been investigated recently. We will

modify the design of extended LDPC codes by a novel approach for cooperative diversity scheme. Furthermore, in this work, we will compare the coded cooperative diversity using punctured LDPC codes with the new design of extended LDPC codes.

The throughput efficiency for the cooperative diversity scheme without any feedback remains constant. The throughput efficiency can be increased with limited feedback from the destination to the users. In this work, new acknowledgment/negative-acknowledgement (ACK/NACK) based protocols will also be proposed for the extended LDPC coded cooperative diversity scheme with improved throughput efficiency as compared to non-acknowledgement based coded cooperative diversity scheme.

Keywords: *LDPC codes, punctured LDPC codes, extended LDPC codes, cooperative diversity.*

مُلخّص الرسالة

الاسم الكامل: حسين علي

عنوان الدراسة: شفرة فحص الكفاءة منخفضة الكثافة لمخططات التنويع التعاوني المعتمدة على طلب إعادة الإرسال الأتوماتيكي

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تستخدم تقنيات التنويع للتخفيف من تشويه وتأثيرات قنوات الاتصالات اللاسلكية. التنويع التعاوني يساعد في زيادة كسب التنويع دون إضافة هوائيات مادية للمستخدمين أو محطات للأجهزة الجوالة. في هذا العمل، يتم استخدام التنويع التعاوني كإطار رئيسي.

لقد تم توسيع التنويع التعاوني إلى التنويع التعاوني المشفر مع إضافة شفرة (RCPC). إن الرموز منخفضة كثافة فحص التكافوء (LPDC) لديها قدرة أفضل لتصحيح الأخطاء من شفرة (RCPC). ولذلك، يمكن أن تكون متكاملة مع مخطط شفرة (LPDC) ذات التنويع التعاوني لزيادة مكاسب التنويع. في الآونة الأخيرة تم التحقيق والبحث في شفرة (RCPC) الموسعة وشفرة (LPDC) على حد سواء. سوف نقوم بتعديل تصميم شفرة (LPDC) الموسعة بواسطة نهج جديد لنظام التنوسع التعاوني. بل وعلاوة على ذلك، في هذا العمل، سوف نقارن بين التصميم الجديد من شفرة (LPDC) مع شفرة التنويع التعاوني الموسع باستخدام شفرة (LPDC) المثقوبة.

كفاءة الإنتاجية لمخطط التنويع التعاوني دون أي إجابات لا تزال كما هي. يمكن زيادة كفاءة الإنتاجية مع إجابات محدودة من الوجهة إلى لمستخدمين. في هذا العمل، تم اقتراح بروتوكول جديد للاعتراف الإيجابي و والاعتراف السلبي (ACK/NACK) وذلك لمخطط شفرة (LPDC) التعاونية الموسعة لتحسن كفاءة الإنتاجية بالمقارنة مع شفرات التنويع التعاونية التي لا تعتمد على الاعتراف.

Nomenclature

Abbreviations

ACK	:	Acknowledgement
APP	:	A Posteriori Probability
ARQ	:	Automatic-Repeat-Request
AWGN	:	Additive White Gaussian Noise
BER	:	Bit Error Rate
BPSK	:	Binary Phase-Shift Keying
CDMA	:	Code Division Multiple Access
CN	:	Check Node
CRC	:	Cyclic Redundancy Check
DF	:	Decode-and-Forward
FDMA	:	Frequency Division Multiple Access
FER	:	Frame Error Rate
LDPC	:	Low-Density Parity-Check codes
LLR	:	Log-Likelihood Ratio
RCPC	:	Rate-Compatible Punctured-Convolutional codes
MAC	:	Multiple Access mode
MIMO	:	Multiple-Input Multiple-Output
MISO	:	Multiple-Input Single-Output

MPA	:	Message-Passing Algorithm
MRC	:	Maximal-Ratio-Combining
NACK	:	Negative-Acknowledgement
SNR	:	Signal-to-Noise Ratio
SPA	:	Sum-Product Algorithm
TDMA	:	Time Division Multiple Access
VN	:	Variable Node

Notations

k	:	Number of information bits
m	:	Number of rows in $\mathbf{H}$ matrix
n	:	Number of bits in a codeword
		Number of columns in $\mathbf{H}$ matrix
$\mathbf{H}$	:	Parity check matrix
w_r	:	Row weight of $\mathbf{H}$ matrix
w_c	:	Column weight of $\mathbf{H}$ matrix
R	:	Code rate
d_{min}	:	Minimum distance of the code
$\mathbb{F}_2$	:	Galois field GF(2)
C	:	Block code ensemble
$\mathbf{G}$	:	Generator matrix
$\mathbf{I}$	:	Identity matrix
$\mathbf{P}$	:	Parity matrix part in $\mathbf{G}$
$\mathbf{O}$	:	Matrix with all-zero entries
$\mathbf{v}$	:	Transmitted codeword
$\hat{\mathbf{v}}$	:	Estimate of received codeword
$\mathbf{y}$	:	Received codeword from the channel
$\mathbf{x}$	:	BPSK modulated codeword

σ^2 : Noise variance

η : Additive white Gaussian noise

α : Fading channel coefficient

L : Log-likelihood ratio

p_{bits} : Number of punctured bits

e_{bits} : Number of extended bits

i : Information bits

p : Parity bits

n' : Size of extended codeword

T_u : Terminal u

N_r : Codeword r

$N_r^{T_u}$: Codeword r transmitted by terminal u

$[\,.\,]^{\mathrm{T}}$: Transpose of a matrix

$\mathbb{N}$: Duration of one frame

CHAPTER 1

INTRODUCTION

1.1 Background

1.1.1 Wireless Communications

It has been more than a century since the first use of wireless telegraphy. During this period, wireless communication have taken many forms in which some has survived but others have faded away with time. The last decade have seen a rapid evolution and development of the mobile wireless communication. The challenge that still hold is to achieve the capacity of the wireless channels that are still poorly defined. Wireless channels suffer from severe degradation due to fading and other forms of channel impairments. The current research in this area is entirely focused on achieving the channel capacity with the best utilization of the channel bandwidth and available resources.

1.1.2 Error Correcting Coding

The channel impairments cause errors in the information transmitted over wireless channels. Error correction coding is applied to detect and correct the errors occurring during transmission and reception by adding some redundant information. The redundant information is called parity check which validates the integrity of the data received. The decoding algorithms use this redundant information to correct the errors to some extent. Every error correcting code has some limitation on its error detection and correction capability. The capability of the code generally increases with more redundant information. The channel efficiency reduces with every single extra parity bit added to the original information. Hence, the researchers have always been in search for the best codes with optimum error correction capability within available resources [1]–[3].

The two main types of error correcting codes are *block codes* and *convolutional codes*. Block codes are further divided into two categories, *linear* and *non-linear* block codes. Convolutional codes [4] are different in structure as compared to block codes. The Viterbi algorithm [5] is used for the decoding of convolutional codes.

Low-Density Parity-Check Codes

Low-density parity-check (LDPC) codes were invented by Gallager in his Ph.D. work [6] in 1960. LDPC codes belong to the class of linear block codes. These codes were ignored due to lack of appropriate hardware in 1960s. A graphical representation of LDPC codes was proposed by Tanner [7] in 1981 based on bipartite graphs. These codes were rediscovered by MacKay [8], [9] and others [10], [11] in 1990s. These codes have become more practical due to the advancements in transistor technology leading to high computational power of the hardware. These codes have gained attention due to their near-capacity performance.

1.1.3 Diversity

Diversity is one of the techniques to combat channel fading. Diversity has been under study in different types. Time diversity, frequency diversity and spatial diversity have gained popularity in wireless communication.

Time Diversity

In *time diversity*, it is desirable to spread the error in time so that error correction techniques can be applied on uncorrelated data. The simplest time diversity scheme is repetition coding in which the data or codeword is retransmitted after some specific time interval. Interleaving is usually used to spread the error in time and reduce the correlation of the channel fade [12], [13].

Frequency Diversity

If the data is transmitted over several channels separated in frequency, we call this form of diversity *frequency diversity*. More specifically for mobile wireless channels, the signals received from multipaths are resolved at the receiver to achieve frequency diversity or multipath diversity [12], [13].

Spatial Diversity

Spatial diversity, also known as *antenna diversity* or *space diversity*, is achieved by placing multiple antennas at the transmitter (transmit diversity) [14], [15] or at the receiver (receive diversity) [16] or at both locations (multiple-input multiple-output systems) [17] separated in space.

Cooperative Diversity Cooperative diversity is a special case of spatial diversity. The mobile terminals are mostly equipped with single transmitting antenna. However, the antennas of mobile terminals can be shared to create a virtual transmit diversity, called *relay diversity or cooperative diversity.* The capacity of the three terminal relay channel was investigated in [18].

Cooperative diversity or user cooperation diversity has been used to achieve diversity gain using the partners transmitting antennas [19], [20]. If the channel with one user to the destination is bad, then the channels from other users, called partners, can be used to send the packet to the destination. The destination receives multiple packets of the same data from independent channels that may not be noisy or in deep fade at the same time. The destination provides decoding by maximal ratio combining on the packets received and thus achieving spatial diversity gain in simple repetition schemes. In a relay channel, each user acts only as relay, i.e. it only forwards the data which it receives by employing either detect-and-forward or amplify-and-forward or estimate-and-forward techniques [21]. In cooperative communication, each user sends its own data as well as relays the data of the partner in different time slots. All the users have single transmitting antenna but they cooperatively create virtual transmit diversity.

1.2 Literature Survey

Wireless communications face the challenges of channel impairments and fading that severely degrade the capacity of wireless channels. Numerous spatial diversity techniques have been in use to combat channel impairments and fading. One such technique is cooperative diversity in which the users or mobile stations cooperate in a particular scenario to exploit the availability of good channels from users to base station or destination. In cooperative diversity, generally, the destination receives multiple packets for the same data from independent channels creating a virtual transmit diversity. Cooperative diversity cannot guarantee error free transmission, therefore, error control coding techniques are applied in cooperative scenario. Low-density parity-check (LDPC) codes have been popular for their capacity-achieving performance. In this work, we will carry out further investigation on the performance of LDPC codes in cooperative diversity.

1.2.1 LDPC Codes

LDPC codes are classified into two broad categories, regular LDPC codes and irregular LDPC codes. In regular LDPC codes, both the column weight and the row weight of the parity-check matrix $\mathbf{H}$ are constant. In irregular LDPC codes, either the column weight or the row weight or both are not constant in $\mathbf{H}$ matrix. Gallager's work was based on regular LDPC codes design [6], [22]. MacKay proposed a semi-random construction algorithm for the **H** matrix of irregular LDPC codes [8], [9]. Later on, Richardson et. al. [23] proposed an optimized design for irregular LDPC codes and proved that these codes can approach the Shannon limit within 0.0045 dB. They introduced the density evolution algorithm for the optimization of these codes. A survey on the design of LDPC codes was presented in [24]. A comprehensive literature on the research work done until 2009 on the various designs of LDPC codes has been published in [3].

The encoding of LDPC codes requires Gauss-Jordan elimination to put the **H** matrix in its systematic form. An efficient encoding algorithm was proposed in [25] by putting the **H** matrix in lower triangular form and making the conversion to systematic form with less computations. Other encoding algorithms have been presented in [26], [27].

LDPC codes are decoded iteratively. Gallager, in his work, proposed a decoding algorithm, also known as sum-product algorithm (SPA) in literature. The log-domain version of SPA, known as log-SPA, replaces multiplications with additions, hence making the log-SPA less complex and more numerically stable in computations than the probability-domain SPA decoder without any performance degradation. Another variation of the SPA is the reduced complexity min-sum algorithm [28]. The reduced complexity min-sum decoder has less computational complexity but suffers from performance degradation [29].

Rate-Compatible LDPC Codes

Rate-compatible punctured codes were introduced in [30]–[32]. The idea of achieving higher code rates by puncturing the lower rate mother code for the convolutional codes was investigated in [32]. The effect of puncturing on turbo codes was investigated in [33]–[35]. Punctured and extended LDPC codes have been investigated in literature to achieve rate compatibility [36]–[41]. Effect of puncturing on LDPC codes was studied in [38]–[40]. Good puncturing patterns were searched in [38] for very large block sizes. Random puncturing and intentional puncturing were compared in [40] for short length block sizes. They showed in their work that intentional puncturing outperforms random puncturing and there exist puncturing distributions that have near optimum performance for a particular code rate. However, at higher rates puncturing causes performance loss due to large amount of erasures inserted at punctured location for soft iterative decoding. Extended codes were studied in [37], [41].The joint design of rate-compatible LDPC codes with both extending and puncturing is investigated in [36], [37]. The design of J. Li et.al. [36] was based on regular LDPC codes whereas M. R. Yazdani et.al. [37] designed the extended LDPC codes based on irregular design by progressive edge growth algorithm. The performance of the extended code designed by M. R. Yazdani et.al. was better than the design of J. Li. et.al.

The joint design of puncturing and extending of regular LDPC codes discussed in [36] is preferred in ARQ protocols for its rate adaptability and can be used in cooperative diversity scheme. In this design, the **H** matrix was extended by padding additional rows and columns leading to lower code rates. The extended **H** matrix was designed in such a way that the parity bits for the higher rate codeword are embedded in the lower rate codeword. The extended **H** matrix design presented in [36] suffers performance loss because the padded rows and columns are very sparse. Due to lesser number of ones in the extended part of **H** matrix, the error corrections of codewords is not improved to a greater extent. However, the complexity in extended codes is marginally decreased because to decode the higher rate codeword, the decoder only needs the non-extended parity-check matrix. Puncturing is done to achieve higher code rates in their work. Punctured LDPC codes are decoded with the insertion of erasures at the punctured locations.

1.2.2 Coded Cooperative Diversity

In coded cooperative diversity or cooperation diversity through coding [42], channel coding is applied jointly with cooperation. The three relay protocols; decode-and-forward (DF), estimate-and-forward and amplify-and-forward have been discussed with channel coding in [43]. Coded cooperative diversity with rate-compatible punctured convolutional (RCPC) codes was introduced in [42], [44]. Turbo codes along with space-time transmission as an extension to coded cooperative diversity scheme were studied in [45].

A high diversity gain has been observed using rate-compatible punctured convolutional (RCPC) codes in cooperative diversity. In the work of T. E. Hunter et.al. [42], [44], the data block is converted to a codeword using convolutional codes. In the cooperative transmission protocol with two users, the codeword at each user is broken into two codewords by periodic puncturing patterns introduced in [32]. Both punctured codewords have the capability of complete information recovery. The first codeword is broadcasted by the user to its partner and to the destination. Under good channel condition between the user and the partner, the first codeword of the user is successfully received by the partner. The second codeword is then relayed by the partner to the destination. At the destination, the receiver decodes the first codeword with erasures inserted at punctured locations. If the codeword is error free, then the second codeword is discarded. If the codeword is not error free, then the second codeword which is received from an independent channel, is decoded with erasures inserted at punctured locations. If the codeword is still not error free, then the two codewords are concatenated together and jointly decoded. So, the decoding at the receiver is done in three steps. The channel model used in their work is block fading (very slow fading) channel.

LDPC-coded Cooperative Diversity

LDPC codes [22] are capacity approaching block codes [9], [23]. The complexity of LDPC codes lies in iterative decoding of blocks of large size. Due to their good error correction capabilities, LDPC codes can be used in relay channels [46]–[48] and cooperative relays [49].

The idea of puncturing the codewords to be used in cooperative diversity scheme was floated in [44], [42]. The system model discussed in [44], [42] was used with punctured LDPC codes by Faisal Zaheer in [50], [51].

Chuxiang Li et. al. used the rate-compatible design of LDPC codes of [37] to extend the original codeword. They introduced the half-duplex relay protocol in [49] for a single relay. In half-duplex relay protocol, the transmission is divided in two time slots. In the first time slot, the destination and the relay received the packets from the source. This mode of operation is called broadcast mode. In the second time slot, the destination receives signals from both source and the relay. This mode of operation is called multiple access (MAC) mode. The code design for these operations becomes different from the non-cooperative models. In the broadcast mode, the source transmits an LDPC codeword to both the relay and the destination. In MAC mode, the relay and the source sends extra parity bits for the codeword to the destination. In MAC mode, the destination receives two copies of the extra bits that can be combined optimally by maximal-ratio-combining (MRC). The codeword is decoded by successive decoding at the destination. The codeword received in the broadcast mode is decoded first. If the codeword is not error free, then it is decoded jointly with extra parity bits received in the MAC mode. However, in this protocol the extra parity bits of the extended codeword does not constitute the complete codeword and cannot be decoded alone to recover the information.

Factor graph decoding approach of [46] was applied on two different relay protocols for cooperative relay systems in [52] where joint decoding at the receiver was applied after receiving both packets in two consecutive time slots. The puncturing of LDPC codes for relay channels was also studied in [53] in which they further investigated full-duplex and half-duplex relays. Moreover, the relay protocols discussed in the work of [46], [49], [53] are dependent on direct link between source to destination because the information bits within the codeword are sent on this link.

1.2.3 ARQ-based Cooperative Diversity

Relaying protocols are essential for the spectral efficiency and the throughput of the cooperative diversity. In [54], relaying protocols were examined in cooperative network.

Fixed relaying protocol is classified into two categories based on the processing of the data received at the relay. The first one is the amplify-and-forward protocol in which the relay amplifies the power of the signal received and forwards it to the destination. The second fixed relaying protocol is the decode-and-forward protocol in which the data received at the relay is decoded, re-encoded and then sent to the destination.

The protocols that adapt to the channel conditions are called *selection relaying* protocols. In these protocols, the decision of the relaying is based on the channel condition between the cooperating users.

Incremental relaying is based on the feedback from the destination. Incremental relaying is more efficient in the utilization of the channel as compared to fixed relaying and selection relaying.

If a packet is not error free at the destination after error correction, then a retransmission is requested to the source by a negative acknowledgment via feedback channel. This retransmission strategy based on the feedback from the destination is called automatic-repeat-request (ARQ). ARQ is related to the retransmissions of the source to the destination whereas relaying protocols are related to the cooperating partner for the exploitation of spatial diversity.

G. Yu et. al. [55] extended the work of [54] to three new protocols for cooperative ARQ based on transmissions in different time slots. The incremental relaying protocol in coded cooperative diversity with RCPC codes was analyzed in [56]. Study on the throughput of cooperative communication with RCPC codes and punctured LDPC codes based on incremental relaying protocol was further carried out in [57], [58].

1.3 Thesis Contributions

In this thesis, the following objectives are achieved:

- The design of the extended LDPC codes is modified for cooperative diversity schemes with a three-step decoding approach. The BER performance of the proposed design of extended LDPC codes is compared with punctured LDPC codes over AWGN channel.

- The proposed extended LDPC codes design is compared with punctured LDPC codes in cooperative diversity scheme. The benchmarks for comparison is BER/FER. We have further analyzed and compared encoding/decoding complexity for the punctured and extended LDPC-coded cooperative diversity scheme.

- The non-feedback-based extended LDPC-coded cooperative diversity scheme is extended to two new feedback-based cooperative diversity protocols. We compare the enhancement in the average throughput of the feedback-based cooperative diversity with non-feedback-based cooperative diversity scheme. We also compare the performance of both proposed protocols on the basis of throughput and FER.

1.4 Thesis Layout

The thesis is organized as follows: In Chapter 2, we revisit the construction of low-density parity-check matrix for LDPC codes and encoding/decoding of LDPC codes. We also propose the new modification to extended LDPC codes. Punctured LDPC codes are also discussed and they are used for comparison with extended LDPC codes. The BER performance of these codes is investigated in AWGN and uncorrelated Rayleigh flat fading channels.

In Chapter 3, the modification to the design of extended LDPC codes is integrated with cooperative diversity. We have analyzed the BER and FER performance of extended and punctured LDPC codes with different inter-user channel SNR. We also discuss the encoding/decoding complexity of extended LDPC-coded cooperative diversity.

In Chapter 4, we propose two new feedback-based coded cooperative diversity protocols with extended LDPC codes and analyze the throughout efficiency of these protocols.

The thesis is concluded with future work in this area in Chapter 5.

CHAPTER 2

LOW-DENSITY PARITY-CHECK CODES

2.1 Introduction

In this chapter, we will revisit the LDPC codes in accordance with Tanner graphs [7] in section 2.2. We will discuss the semi-random construction [8], [9] in section 2.2.2 and encoding of these codes in section 2.3. The probability-domain and log-domain decoding algorithms for these codes will also be discussed in section 2.4.

Rate-compatible LDPC codes will be constructed by puncturing and extension in sections 2.5.1 and 2.5.2 respectively. We will propose a modification to extended LDPC codes in section 2.5.3 which will be useful in cooperative diversity schemes (to be discussed in Chapter 3). The bit error rate performance of these codes will be analyzed in AWGN and uncorrelated Rayleigh fading channels in section 2.6.

2.2 Construction of LDPC Codes

An LDPC code is defined by the parity-check matrix $\mathbf{H}$ of size $m \times n$ with low density of ones. A *regular LDPC code* has a parity-check matrix $\mathbf{H}$ with constant column weight w_c and constant row weight w_r which are related by $w_r = w_c \times (n/m)$ and $w_c \ll m$ for the $\mathbf{H}$ matrix to be sparse. The LDPC code design presented in Gallager work belongs to the class of regular LDPC codes. For good error correction capability, $w_c \geq 3$ [22]. An *irregular LDPC code* has a parity-check matrix in which both w_c and w_r are not constant. The irregular LDPC codes have better error correction capabilities and perform better than the regular LDPC codes. The optimum design for irregular LDPC codes has been shown to achieve near-capacity performance [23].

2.2.1 Tanner Graph

A geometrical figure which consists of nodes and edges (lines connecting the nodes) is called a *graph*. In a *bipartite graph*, the set of nodes is divided into two subsets such that all the edges connect nodes of one subset to the nodes of second subset and there is no edge connecting the nodes within one subset. The graphical representation of LDPC codes proposed by Tanner [7] is a bipartite graph which is useful for the description of decoding algorithms. The two types of nodes in a Tanner graph are the *variable nodes* (VNs) and the *check nodes* (CNs). The code ensemble is represented by Tanner graph using its $\mathbf{H}$ matrix. The edge is drawn between CN i and VN j whenever h_{ij} is 1. For an $\mathbf{H}$ matrix of size $m \times n$, there are m CNs and n VNs in a Tanner graph. The Tanner graph for the following **H** matrix:

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \tag{2.1}$$

is shown in Fig. 2.1.

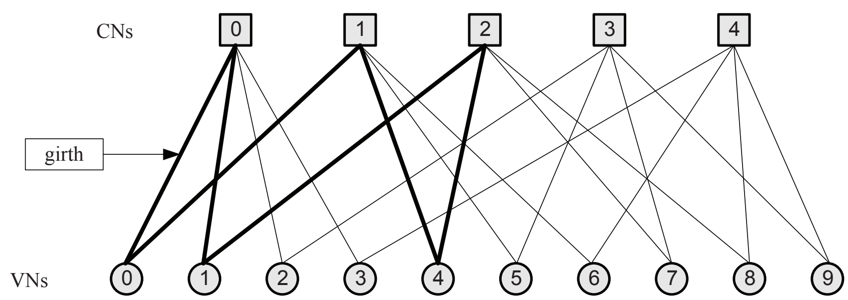


Figure 2.1: Tanner graph.

A *cycle* is a closed path in a Tanner graph that begins and ends at the same node. The length of shortest cycle in a Tanner graph is called *girth*. A girth 6 cycle is highlighted in Fig. 2.1. The girth 4 cycle, called *short cycle*, has a negative impact on the decoding because it reduces the minimum distance d_{min} of the code and therefore they are avoided in the construction of **H** matrix. The minimum distance d_{min} of a code is defined as the minimum number of columns in **H** matrix that adds to zero in $\mathbb{F}_2$. The elimination of short cycle implies that two 1s in a column do not occur at the same position in any other column. This condition is called *row-column* (RC) *constraint*.

2.2.2 Semi-Random Construction

Random construction of **H** matrix is computationally inefficient. Random construction is done by constructing the **H** matrix of size $m \times n$ initially with all zero entries and then randomly flipping the '0' bits to '1'. The random construction algorithm is unable to construct an exact regular **H** matrix with all row weights and column weights constant. The algorithm also cannot guarantee the absence of short cycles. An algorithm with some constraints is preferred to construct **H** matrix with good error correction properties.

A semi-random construction algorithm was proposed by MacKay [8], [9] in which the **H** matrix is constructed under some constraints. An efficient algorithm has been presented in [59] known as bit-filling algorithm for semi-random construction of LDPC codes. The algorithm places the ones in the **H** matrix one-by-one and keep on checking the applied conditions. The bit-filling algorithm has been extended and improved in [60]. This algorithm is computationally fast but destroys the random property of **H** matrix to some extent. In MacKay's algorithm, the RC constraint and constant column weight constraint is applied but the row weight is kept nearly constant. Additionally, the last $n - k$ columns are constrained to be invertible. We also attempt to construct the regular **H** matrix with an additional constraint of constant row weight in this algorithm. The algorithm for the semi-random construction of regular **H** matrix of size $m \times n$ is summarized in Table 2.1.

2.3 Encoding

LDPC codes are defined by their parity-check matrix $\mathbf{H}$ with $\mathbf{G}.\mathbf{H}^T = \mathbf{O}$ where $\mathbf{G}$ is called the generator matrix and $\mathbf{O}$ is an all zero matrix. The $\mathbf{H}$ matrix is put into the systematic form by Gauss-Jordan elimination over $\mathbb{F}_2$. The $\mathbf{H}$ matrix is a *full rank* matrix if the rank of $\mathbf{H}$ is equal to the number of rows of $\mathbf{H}$. For a full rank $\mathbf{H}$ matrix, the code rate for the LDPC code constructed is given by:

$$R = 1 - \frac{m}{n} \tag{2.2}$$

However, in most of the cases the $\mathbf{H}$ matrix constructed is not full rank. The Gauss-Jordan elimination of the $\mathbf{H}$ matrix will result in the following form of $\mathbf{H}$:

$$\mathbf{H} = \begin{bmatrix} \tilde{\mathbf{P}}^{\mathrm{T}} & \mathbf{I} \\ \mathbf{O} & \mathbf{O} \end{bmatrix}_{m \times n} \tag{2.3}$$

where $\tilde{\mathbf{P}}^{\mathrm{T}}$ is the parity part of size $\acute{m} \times (n - \acute{m})$ and $\acute{m} < m$. In this case, we use the matrix:

$$\mathbf{H} = [\ \tilde{\mathbf{P}}^{\mathrm{T}} \quad \mathbf{I}\ ] \tag{2.4}$$

Semi-random construction for regular LDPC
1. Create an all zero $\mathbf{H}$ matrix of size $m \times n$. 2. Initialize the column number $c = 1$ where $1 \leq c \leq n$. 3. Generate a column vector of weight w_c with 1's placed randomly and insert into column c of $\mathbf{H}$ matrix. 4. If row weight of $\mathbf{H} > w_r$, then go to step 3. 5. If there is a short cycle, then go to step 3. 6. If conditions 4 & 5 are satisfied, then increment c and go to step 3. 7. Stop if $c = n$. 8. $\mathbf{H}$ matrix is further constrained so that last m columns must be invertible in $\mathbb{F}_2$.

Table 2.1: Summary of semi-random construction algorithm for regular LDPC codes [9].

which is of size $\acute{m} \times n$ resulting in a higher code rate. The code rate R for the LDPC code is then bounded by:

$$R \geq 1 - \frac{m}{n} \tag{2.5}$$

2.4 Decoding

Gallager, in his work [22], [6], proposed an iterative decoding algorithm which has near optimum performance. This decoding algorithm for LDPC codes is called sum-product algorithm (SPA). The log-domain version of SPA (log-SPA) is more numerically stable as compared to probability-domain SPA. The performance of both decoders is the same. Both algorithms are based on message-passing generic algorithm. We will discuss both algorithms briefly here.

2.4.1 Probability-domain Sum-Product Algorithm

We need to compute the a posteriori probability (APP) that a given bit in the transmitted codeword $\mathbf{v}$ equals 1 given the received codeword $\mathbf{y}$ where $\mathbf{v} = [v_0 \;\, v_1 \ldots v_{n-1}]$ and $\mathbf{y} = [y_0 \;\, y_1 \ldots y_{n-1}]$. To decode bit v_j, we have to compute APP given by,

$$\Pr(v_j = 1|\mathbf{y})$$

so that the APP ratio or the likelihood ratio is

$$l(v_j|\mathbf{y}) = \frac{\Pr(v_j = 0|\mathbf{y})}{\Pr(v_j = 1|\mathbf{y})}. \tag{2.6}$$

For the additive white Gaussian noise (AWGN) channel and binary phase-shift keying (BPSK) or antipodal scheme,

$$y_j = x_j + n_j \tag{2.7}$$

where $x_j = -2v_j + 1 \in \{-1, +1\}$ and n_j are samples from independent and identically distributed (i.i.d.) Gaussian noise with zero mean and variance σ^2. The APP of $x_j = 1$ given $\mathbf{y}$ is received is

$$\begin{aligned} \Pr(x_j = +1|\mathbf{y}) &= \frac{\Pr(\mathbf{y}|x_j = +1)\Pr(x_j = +1)}{\Pr(\mathbf{y})} \\ &= \frac{1}{1 + \exp(-2\mathbf{y}/\sigma^2)} \end{aligned} \tag{2.8}$$

where we have assumed x_j is uniformly distributed with $\Pr(x_j = +1) = \Pr(x_j = -1) = 1/2$, $n_j \sim \mathcal{N}(0, \sigma^2)$ and

$$\Pr(\mathbf{y}) = \frac{1}{2}\Pr(\mathbf{y}|x_j = +1) + \frac{1}{2}\Pr(\mathbf{y}|x_j = -1). \tag{2.9}$$

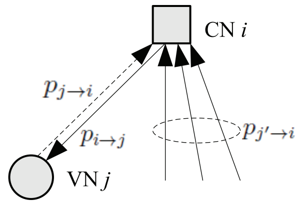


Figure 2.2: CN update for the probability domain decoding.

Similarly,

$$\begin{aligned}\Pr(x_j = -1|\mathbf{y}) &= \frac{\Pr(\mathbf{y}|x_j = -1)\Pr(x_j = -1)}{\Pr(\mathbf{y})} \\ &= \frac{1}{1 + \exp(2\mathbf{y}/\sigma^2)}. \end{aligned} \tag{2.10}$$

In general,

$$\Pr(x_j = x|y_j) = \frac{1}{1 + \exp(-2y_j x/\sigma^2)} \tag{2.11}$$

where $x \in \{+1, -1\}$ and y_j is the jth received value from the channel.

The sum-product algorithm is based on generic message-passing algorithm. In the probability-domain decoding, the message passed among the nodes are the probabilities initialized by Eq. 2.11. The check node i receives the probabilities from all of its connected neighbors $N(i)$ excluding the message $p_{j\to i}$. Figure 2.2 is an illustration of the message passing for CN update.

For CN update $p_{i\to j}$ is given by [22], [3]

$$p_{i\to j}(b) = \frac{1}{2} + \frac{(-1)^b}{2} \prod_{j' \in N(i) - \{j\}} (1 - 2p_{j'\to i}(1)) \tag{2.12}$$

where $N(i)$ is the set of neighbors of i.

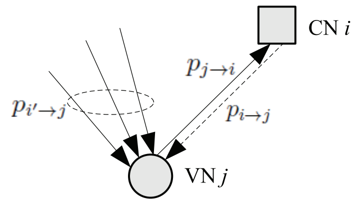


Figure 2.3: VN update for the probability domain decoding.

Figure 2.3 illustrates the message passing for VN update for VN j. VN j receives the information from the neighbors of j excluding $p_{i\to j}$ and sends it as $p_{j\to i}$ to CN i. The equation for $p_{j\to i}$ is given by [22], [3]

$$p_{j\to i}(b) = K_{ji}\Pr(v_j = b|y_j) \prod_{i'\in N(j)-\{i\}} (p_{i'\to j}(b)) \tag{2.13}$$

where $N(j)$ is the set of neighbors of VN j and

$$\Pr(v_j = b|y_j) = \Pr(x_j = x|y_j) \tag{2.14}$$

with $v_j \in \{0,1\}$ and $x \in \{+1,-1\}$.

The algorithm is summarized [3] in Table 2.2. The discussion also holds for Rayleigh fading channels. The Rayleigh fading channel can be modeled as

$$y_j = \alpha_j x_j + n_j \tag{2.15}$$

where α_j are the Rayleigh distributed channel coefficients with unit variance. In

Probability-domain sum-product algorithm

1. **Initialization:** For all i, j for which $h_{ij} = 1$, initialize $p_{j\to i}(0) = \Pr(x_j = +1|y_j)$ and $p_{j\to i}(1) = \Pr(x_j = -1|y_j)$ using Eq. 2.11 or 2.16, where y_j is the jth received channel value.

2. **CN update:** For each $b \in \{0, 1\}$, update $\{p_{i\to j}(b)\}$ at each CN using

$$p_{i\to j}(b) = \frac{1}{2} + \frac{(-1)^b}{2} \prod_{j'\in N(i)-\{j\}} (1 - 2p_{j'\to i}(1)).$$

3. **VN update:** For each $b \in \{0, 1\}$, update $\{p_{j\to i}(b)\}$ for each VN using

$$p_{j\to i}(b) = K_{ji}\Pr(v_j = b|y_j) \prod_{i'\in N(j)-\{i\}} (p_{i'\to j}(b)),$$

where the constants K_{ji} are selected to ensure that $p_{j\to i}(0) + p_{j\to i}(1) = 1$.

4. **Total probability:** For each $b \in \{0, 1\}$, and for each $j = 0, 1, \ldots, n-1$, compute

$$p_j(b) = K_j\Pr(c_j = b|y_j) \prod_{i\in N(j)} (p_{i\to j}(b)),$$

where the constants K_j are chosen to ensure that $p_j(0) + p_j(1) = 1$

5. **Stopping criteria:** For $j = 0, 1, \ldots, n-1$, set

$$\hat{v}_j = \begin{cases} 1 & \text{if } P_j(1) > P_j(0), \\ 0 & \text{otherwise} \end{cases}$$

to obtain $\hat{\mathbf{v}}$. If $\hat{\mathbf{v}}\,\mathbf{H}^{\mathrm{T}} = 0$ or the number of iterations equals the maximum limit stop; else go to Step 2.

Table 2.2: Summary of the probability-domain sum-product algorithm for the decoding of LDPC codes [3].

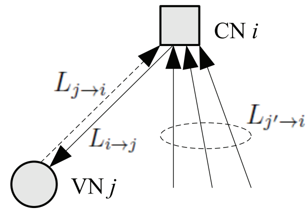


Figure 2.4: CN update for log-domain decoding.

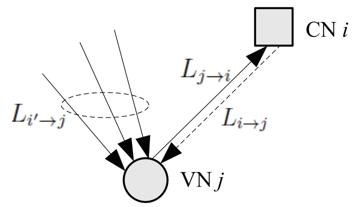


Figure 2.5: VN update for log-domain decoding.

case of Rayleigh fading channel, the decoder is initialized by

$$\Pr(x_j = x|y_j) = \frac{1}{1 + \exp(-2\alpha_j y_j x/\sigma^2)} \tag{2.16}$$

with the assumption that the perfect channel estimates for α_j are known at the receiver.

2.4.2 Log-domain Sum-Product Algorithm

The log-domain version of SPA is more numerically stable. The message passing principle for log-domain decoder is the same as probability-domain decoder except the information passed among the nodes is the log likelihood ratio (LLR). Figure 2.4 and 2.5 illustrates the LLR information passing among the nodes. The LLR is the log of the likelihood ratio and is given by

$$L_j = L(v_j|y_j) = \log\left(\frac{\Pr(v_j = 0|y_j)}{\Pr(v_j = 1|y_j)}\right) \tag{2.17}$$

For additive white Gaussian (AWGN) channels, let $x_j = (-1)^{v_j}$ be the jth transmitted binary value. The jth received sample is $y_j = x_j + n_j$, where the n_j are independent and normally distributed as $\mathcal{N}(0, \sigma^2)$. The decoder is initialized by the following equation

$$L_j = 2y_j/\sigma^2 \tag{2.18}$$

The log-domain SPA is summarized [3] in Table 2.3. For the Rayleigh channel, the decoder is initialized by the following

$$L_j = 2\alpha_j y_j/\sigma^2 \tag{2.19}$$

2.5 Rate-Compatible LDPC Codes

Rate-compatible codes generate codewords in which the codewords from the high rate codes are embedded in the low rate codes. They require a single encoder at the transmitter and a single decoder at the receiver. Rate-compatibility is desired in many applications such as adaptive coding systems and automatic repeat request with forward error correction protocols. These codes help to vary the degree of protection for a particular data block, for example, header of a frame can

The log-domain sum-product algorithm

1. **Initialization:** For all j, initialize by Eq. 2.18 or 2.19. For all $i.j$ for which $h_{ij} = 1$, set $L_{j\to i} = L_j$.

2. **CN update:** Compute outgoing CN messages $L_{i\to j}$ for each CN using

$$L_{i\to j} = 2\tanh^{-1}\left(\prod_{j'\in N(i)-\{j\}} \tanh\left(\frac{1}{2}L_{j'\to i}\right)\right)$$

   and then transmit to the VNs.

3. **VN update:** Compute outgoing VN messages $L_{j\to i}$ for each VN using

$$L_{j\to i} = L_j + \sum_{i'\in N(j)-\{i\}} L_{i'\to j}$$

   and then transmit to the CNs.

4. **LLR total:** For $j = 0, 1, \ldots, n-1$ compute

$$L_j^{\text{total}} = L_j + \sum_{i\in N(j)} L_{i\to j}$$

5. **Stopping criteria:** For $j = 0, 1, \ldots, n-1$, set

$$\hat{v}_j = \begin{cases} 1 & \text{if } L_j^{\text{total}} < 0 \\ 0 & \text{otherwise} \end{cases}$$

   to obtain $\hat{\mathbf{v}}$. If $\hat{\mathbf{v}}\,\mathbf{H}^{\mathrm{T}} = 0$ or the number of iterations equals the maximum limit stop; else go to Step 2.

Table 2.3: Summary of the log-domain sum-product algorithm for the decoding of LDPC codes [3].

be protected with a low rate code while the rest of the frame can be encoded with a high rate code. The rate-compatibility for convolutional codes, turbo codes and BCH codes was achieved successfully. LDPC codes have near capacity performance and can be used with rate-compatibility in practical systems. The rate-compatible LDPC codes are constructed by puncturing or extension methods. We will discuss both methods as follows.

2.5.1 Punctured LDPC Codes

Punctured LDPC codes were studied in [38]–[40] to convert the codes from low rate to high rates. In punctured codes, the bits are punctured according to a predefined pattern. The puncturing pattern can be random or periodic. The effect of puncturing pattern on BER performance for infinite block length was investigated in [38] whereas the puncturing patterns for short block lengths were studied in [39], [40]. In this work, puncturing is done periodically to achieve higher rate codes. The code rate for a punctured code is $R' = k/n'$ which is obtained by puncturing the mother code of rate $R = k/n$. The number of bits punctured is $p_{bits} = n - n'$ where n' is the length of the punctured codeword. The puncturing period to puncture the mother code is n/p_{bits}. At the decoder, erasures are inserted at the punctured positions. An erasure is an unbiased value which is zero in soft decoding.

2.5.2 Extended LDPC Codes

Extended LDPC codes were introduced in [36], [37], [41] to achieve lower rate codes from high rate codes. The design of extended codes proposed in [36], [37], [41] is also capable of embedding higher rate codewords in lower rate codewords. The study conducted in [36] was limited to regular LDPC codes but it was extended to irregular codes in [37], [41].

We will briefly discuss the design of [36]. The extended **H** matrix is designed according to the following definitions of matrices

$$\mathbf{H}_2 = \begin{bmatrix} \mathbf{H}_1 & \mathbf{O} \\ \mathbf{A} & \mathbf{B} \end{bmatrix}_{m' \times n'} \tag{2.20}$$

where $\mathbf{H}_1$ is the matrix of dimensions $m \times n$ for the mother code of rate $R = k/n$. To extend the code rate to $R' = k/n'$, the extra parity bits in the extended codewords will be $e_{bits} = n' - n$ where n' is the size of extended codeword for rate R'. The **O** matrix is an all zero matrix of size $k \times e_{bits}$ or $k \times (n' - n)$. The **A** matrix is a very sparse matrix of size $(n' - n) \times n$ with at least one 1 in each row. The **B** matrix has dimensions $(n' - n) \times (n' - n)$ with column weight 3.

The systematic form of $\mathbf{H}_1$ matrix is

$$\mathbf{H}_1 = \begin{bmatrix} \mathbf{P}_1^{\mathrm{T}} & \mathbf{I}_{n-k} \end{bmatrix}_{m \times n} \tag{2.21}$$

and for $\mathbf{H}_2$ matrix

$$\mathbf{H}_2 = \begin{bmatrix} \mathbf{P}_1^{\mathrm{T}} & \\ & \mathbf{I}_{n'-k} \\ \mathbf{P}_2^{\mathrm{T}} & \end{bmatrix}_{m' \times n'} \tag{2.22}$$

The $\mathbf{O}$ matrix ensures that the higher rate codewords are embedded in extended lower rate codewords by keeping the integrity of $\mathbf{P}_1^{\mathrm{T}}$. The generator matrix for $\mathbf{H}_1$ and $\mathbf{H}_2$ becomes

$$\mathbf{G}_1 = \begin{bmatrix} \mathbf{I}_k & \mathbf{P}_1 \end{bmatrix}_{k \times n} \tag{2.23}$$

and

$$\mathbf{G}_2 = \begin{bmatrix} \mathbf{I}_k & \mathbf{P}_1 & \mathbf{P}_2 \end{bmatrix}_{k \times n'} \tag{2.24}$$

respectively, where $\mathbf{P}_1$ has dimensions $k \times (n-k)$ and $\mathbf{P}_2$ has dimensions $k \times (n'-n)$. The codewords are generated using $\mathbf{G}_1$ and $\mathbf{G}_2$ with identical n bits.

2.5.3 Modification to Extended LDPC Codes

We propose a novel modification to the extended LDPC codes. The main objective of this modification is to create codewords of equal lengths from $\mathbf{H}$ matrices of different dimensions while keeping the integrity of information bits in the codewords. The usefulness of this modification will become more evident in Chapter 3 where we will apply this modification to cooperative diversity scheme.

As discussed in section 2.5.2, the codewords generated from $\mathbf{G}_1$ and $\mathbf{G}_2$ have unequal lengths, that is, n and n' ($n' \neq n$) respectively. We modify the codeword generated by $\mathbf{G}_2$ to length n and investigate the effect of this modification on BER performance. The codeword $\mathbf{v}_1$ is generated by using the generator matrix obtained from $\mathbf{H}_1$. Using Equation (2.23), $\mathbf{v}_1$ takes the following form

$$\mathbf{v}_1 = [i \quad p_1]_{1\times n} \tag{2.25}$$

where i is the information part and p is the parity part in the codeword $\mathbf{v}_1$. The second codeword is generated by the generator matrix $\mathbf{G}_2$ mentioned in Equation (2.24), in the following form. We call this codeword $\mathbf{v}_3$.

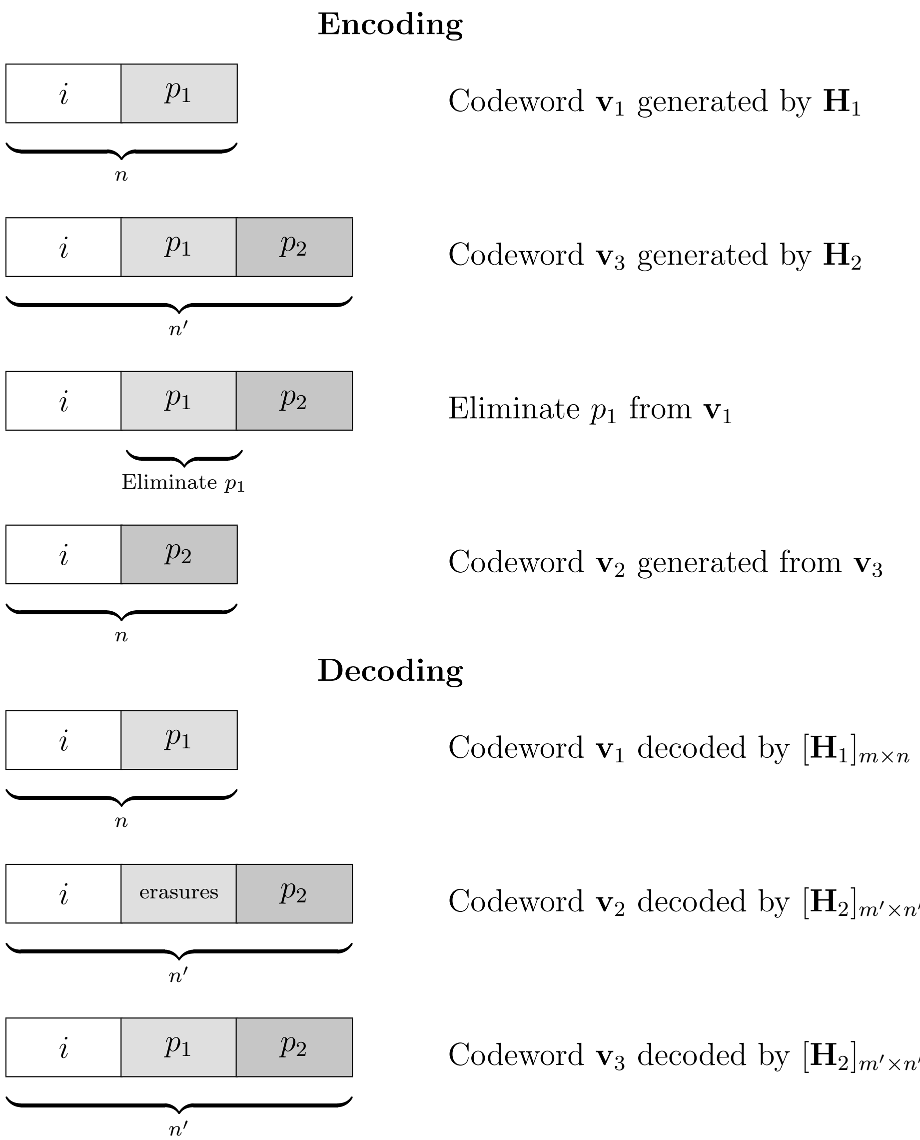
$$\mathbf{v}_3 = [i \quad p_1 \quad p_2]_{1\times n'} \tag{2.26}$$

where p_1 is the same parity part as in $\mathbf{v}_1$ and p_2 is the extended parity part. The codeword $\mathbf{v}_3$ has length n' where $n' > n$. The codeword $\mathbf{v}_3$ is modified to generate a codeword of length n. We call this codeword $\mathbf{v}_2$.

$$\mathbf{v}_2 = [i \quad p_2]_{1\times n}. \tag{2.27}$$

The modification to extended LDPC codes is summarized below.

We will analyze the BER performance of the three codewords $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$. At the receiver, the codeword $\mathbf{v}_1$ is decoded using $\mathbf{H}_1$. The codeword $\mathbf{v}_2$ is decoded using $\mathbf{H}_2$ matrix with erasures inserted equal to the length of p_1 to make the codeword of length n'. The same information i can be recovered from the codeword $\mathbf{v}_3$ that is formed by the concatenation of $\mathbf{v}_1$ and $\mathbf{v}_2$ and decoding it jointly using $\mathbf{H}_2$. The decoding is summarized below.



2.6 Simulation Results and Discussion

We assume antipodal schemes for all simulations. Figure 2.6 shows the BER performance of a (3,6) regular code over AWGN channel. The noise variance is given by $\sigma^2 = 1/(2R \times (E_b/N_0))$ [61] for AWGN channels. The coding gain for the (3,6) regular code with block length $n = 504$ is approximately 4.4 dB at a bit-error probability of 10^{-3} for the AWGN channel.

Figure 2.7 shows the BER performance of the (3,6) regular code over uncorrelated Rayleigh fading channel. A coding gain of approximately 18 dB can be seen at a bit-error probability of 10^{-3} for a (3,6) regular code of block length $n = 512$ over uncorrelated Rayleigh fading channel. The average received SNR per bit for the Rayleigh fading channel is $\overline{\rho}_b = E_b/N_0 \times E(\alpha^2)$ where α is Rayleigh distributed channel coefficient [61].

Figure 2.8 shows the effect of SPA decoder iterations. The codewords converge at nearly 100 iterations. Figure 2.9 and 2.10 show the effect of block length on BER with three different block sizes of $n = 512, 1024, 2048$ over AWGN channel and uncorrelated Rayleigh fading channel respectively. The performance of LDPC codes improves with the increase in the size of the block length because the minimum distance of and LDPC code is a function of block length.

For puncturing of LDPC codes, we choose the (3,6) regular mother code of $R = 1/2$ and block length $n = 2048$. The number of bits punctured from the mother code are 128, 256, 512 to obtain the codes with higher rates 8/15, 8/14, 8/12 respectively. The BER performance of these codes over AWGN channel is shown in Figure 2.11. Since the comparison is made between LDPC codes only, therefore, we assume $\sigma^2 = 1/(2 \times (E_s/N_0))$ for Figure 2.11 and onwards.

For extended codes, we choose a mother code of rate 1/2 of block length $n = 1024$. This code is extended to rates $8/18, 8/20, 8/22, 8/24$ with $e_{bits} = 64, 128, 384, 512$ respectively. Figure 2.12 shows the BER performance for these extended LDPC codes over AWGN channel with $\sigma^2 = 1/(2 \times (E_s/N_0))$. The BER improves with the addition of extra parity bits. With the addition of extra bits, the **H** matrix size is also increased resulting in better BER performance.

Figure 2.13 shows a useful comparison between punctured and extended LDPC codes. An attempt has been made to construct code rate 8/24 both by puncturing and extension while keeping the number of information bits, $k = 512$, constant for all code rates. A regular (3,6) code of rate 8/16 has been extended to 8/24 whereas a regular code of rate 8/32 has been punctured to get a code rate of 8/24. An unaltered regular code of rate 8/24 has also been simulated and plotted for comparison. The BER performance of these codes show that the 8/24 code obtained by extension performs better than the 8/24 code obtained by puncturing. The unaltered 8/24 code performs better than both the codes, either obtained by puncturing or extension. The low rate extended codes obtained by extension of a mother code performs better than the codes obtained by puncturing. In this work, we have presented only one mother code for this comparison, however, this is true for any mother code. For more detailed discussion, the reader is referred to [36], [62].

We compare the modification to extended LDPC codes discussed in section 2.5.3 with punctured LDPC codes. For this comparison, we keep the information bits and codeword lengths transmitted over the channel to be constant. A regular code P-3 with $k = 512$ and $n = 2048$ is constructed. This code is punctured periodically to obtain two codewords of length 1024. We call these two codes as P-1 and P-2. For the modified extended LDPC codes, we define three codes E-1, E-2 and E-3 for the codewords $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ (section 2.5.3) respectively. The code

	Information bits	Codeword length	Size of $\mathbf{H}$ matrix
Code P-1	512	1024	1536×2048
Code P-2	512	1024	1536×2048
Code P-3	512	2048	1536×2048
Code E-1	512	1024	512×1024
Code E-2	512	1024	1024×1536
Code E-3	512	1536	1024×1536

Table 2.4: Description for codes P-1, P-2, P-3, E-1, E-2 and E-3.

E-1 and E-2 has $k = 512$ and $n = 1024$ and are obtained according to Equations (2.25) and (2.27). The code E-3 is obtained according to Equation (2.26) with codeword length 1536. Further details related to codeword lengths and size of $\mathbf{H}$ matrices used for encoding and decoding of these codes have been tabulated in Table 2.4. The simulation results for these codes are shown in Figure 2.14. The codes P-1 and P-2 are rate 1/2 codes obtained by puncturing the mother code of rate 1/4, therefore, they have almost the same BER performance. Code P-3 is obtained by the concatenation of codewords of P-1 and P-2, resulting in an overall code rate of 1/4 and have much better performance than both P-1 and P-2. The code E-1 performs approximately 1.5 dB better than code E-2 at a BER of 10^{-3} over AWGN channel. The code E-2 is decoded by $\mathbf{H}_2$ (Equation 2.20) with erasure insertion which causes the performance loss. The code E-3 is constructed by the concatenation of the codewords of E-1 and E-2. The code E-3 performs approximately 0.4 dB worse than code P-3. This performance loss is the result of forcing the zero in the upper right part of $\mathbf{H}_2$ (Equation 2.20) which alters the random properties of **H** matrix. The code P-3 has the best performance among all these codes because it is decoded on random matrix of size 1536×2048. The usefulness of this modification to extended codes will become more evident in Chapter 3.

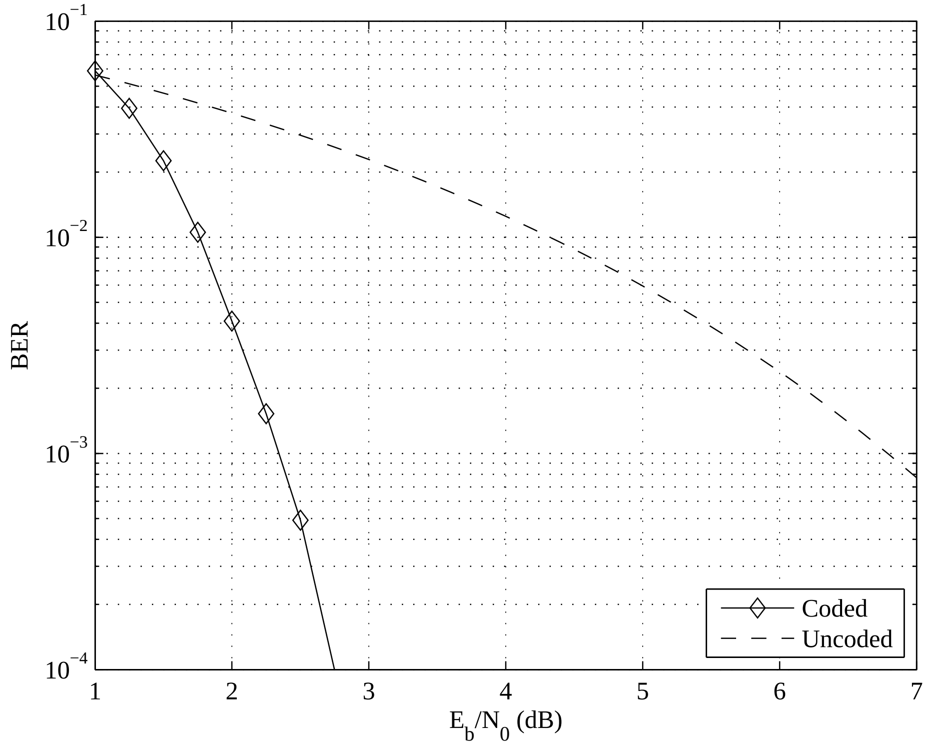


Figure 2.6: BER performance of (3,6) regular code of block length 504 over AWGN channel.

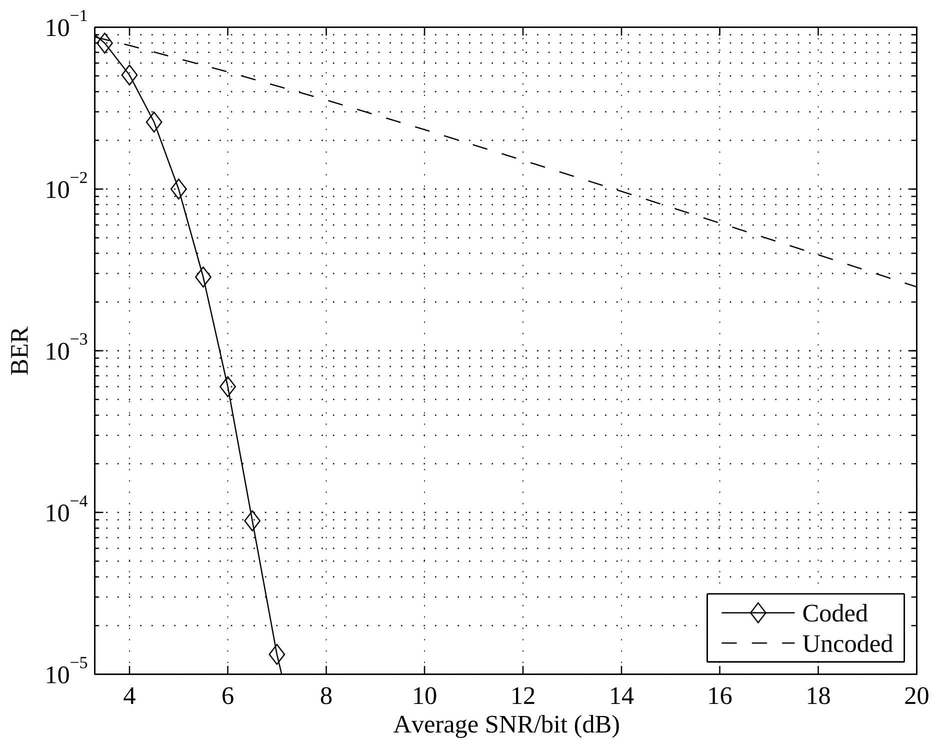


Figure 2.7: BER performance of (3,6) regular code of block length 512 over uncorrelated Rayleigh channel.

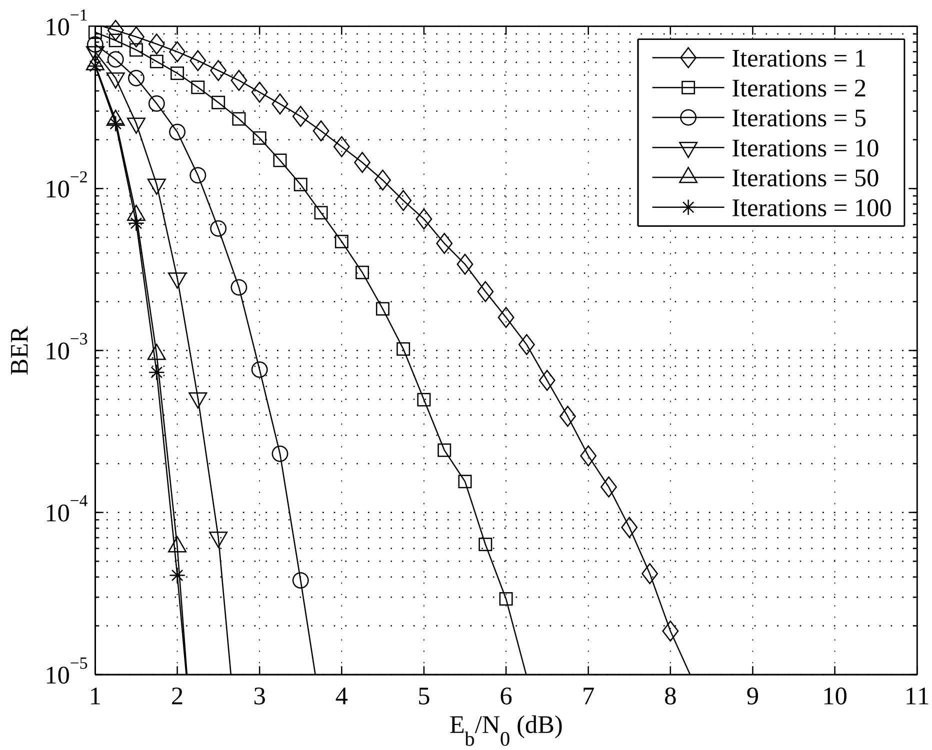


Figure 2.8: Effect of decoder iterations on BER performance of (3,6) regular code of block length $n = 2048$ over AWGN channel.

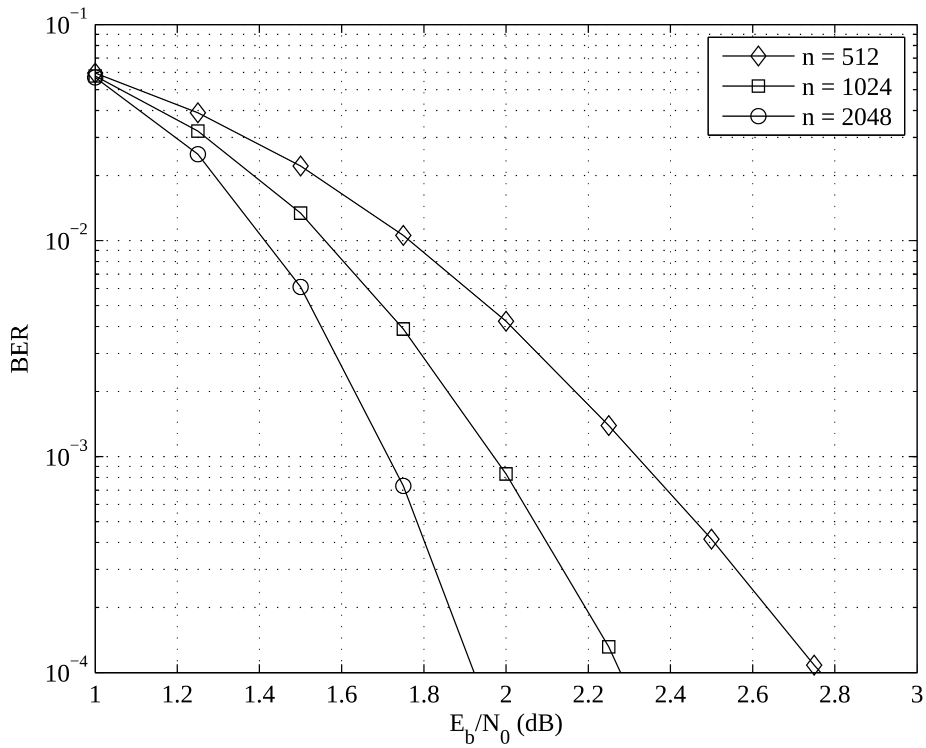


Figure 2.9: Effect of block length on BER performance of (3,6) regular code over AWGN channel.

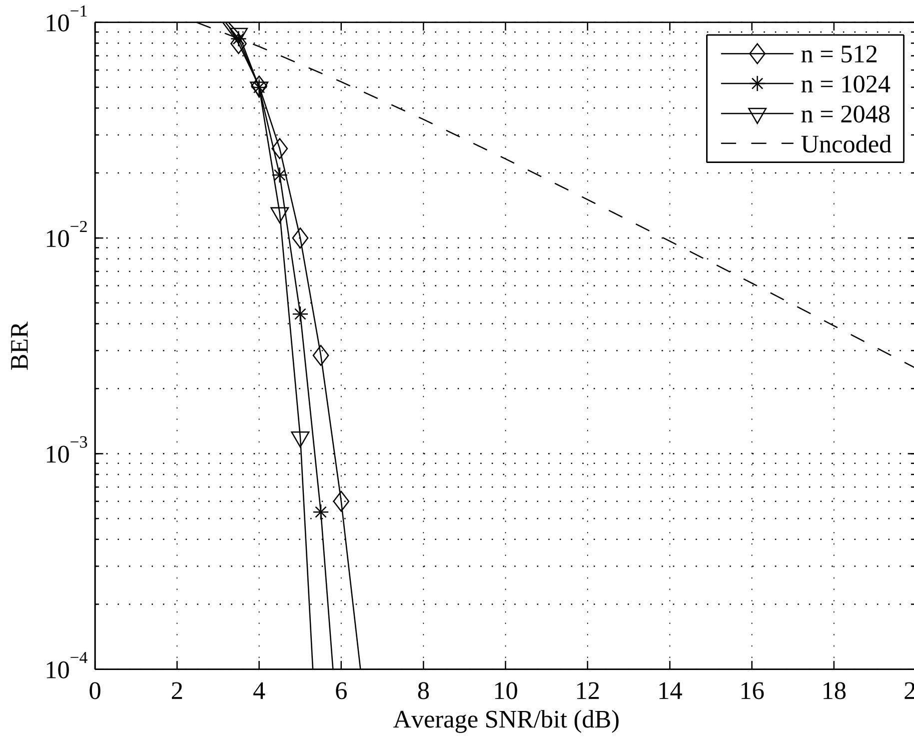


Figure 2.10: Effect of block length on BER performance of (3,6) regular code over uncorrelated Rayleigh fading channel.

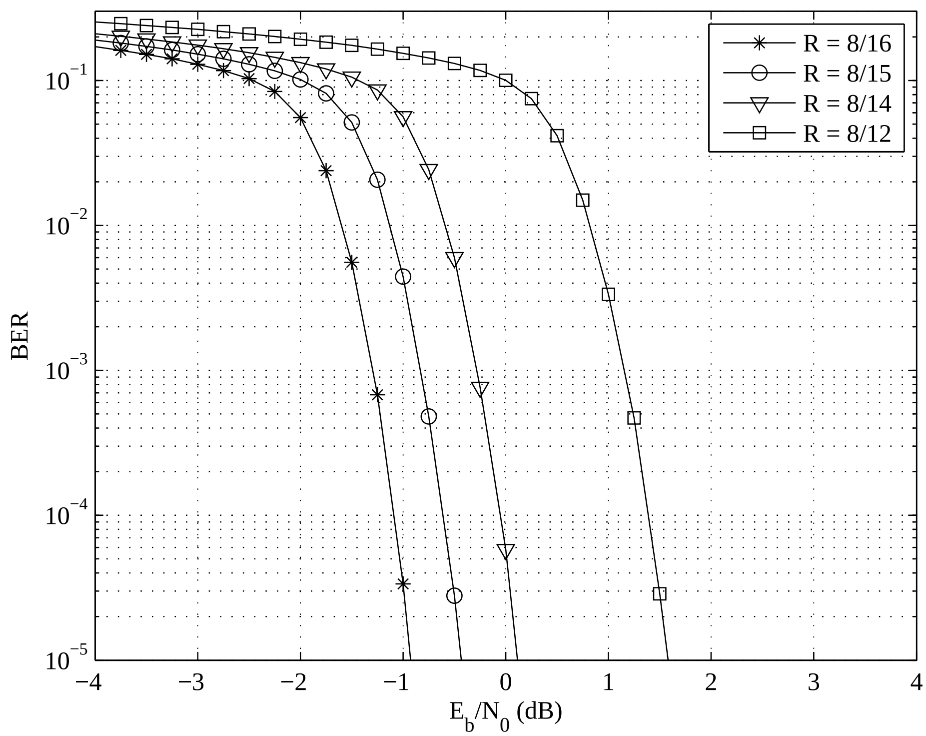


Figure 2.11: Effect of puncturing on regular (3,6) mother code of rate 8/16 and information block length $k = 1024$ over AWGN channel. Code rates obtained after puncturing are 8/15, 8/14 and 8/12.

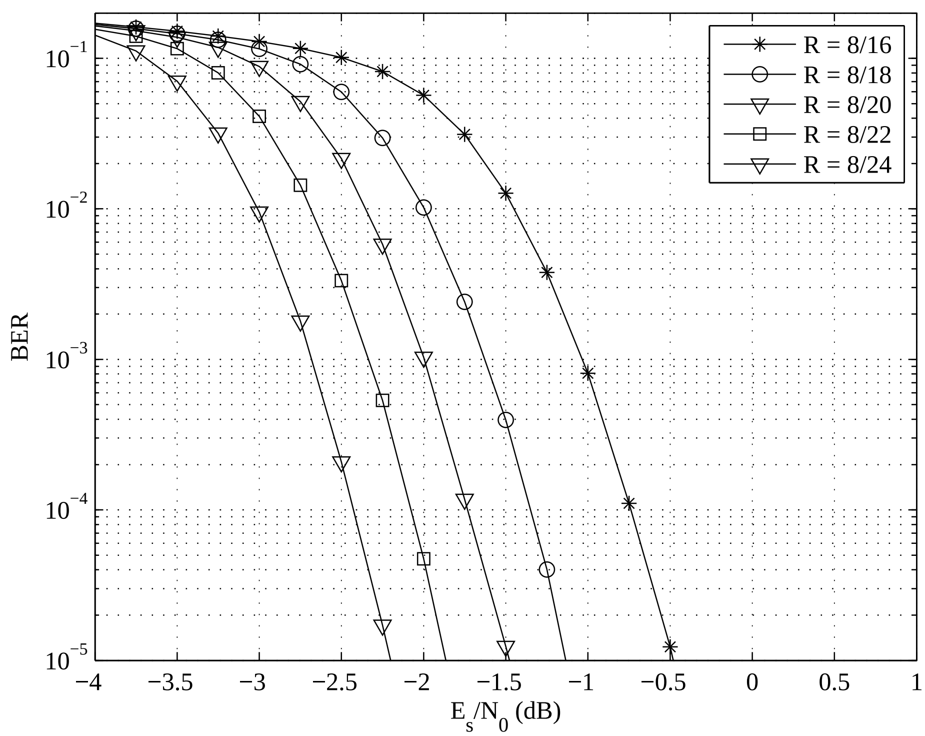


Figure 2.12: BER performance of extended codes. Regular (3,6) mother code of rate of $R = 8/16$ with extension to $8/18, 8/20, 8/22$ and $8/24$ over AWGN channel. Information length $k = 512$.

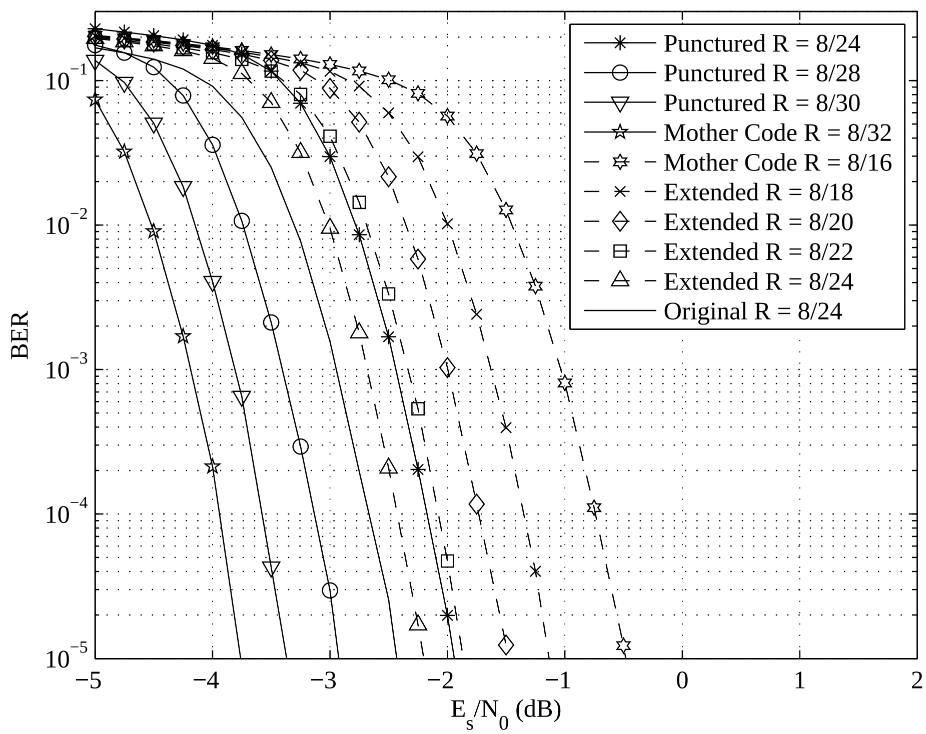


Figure 2.13: BER performance of extended and punctured codes over AWGN channel. Information length $k = 512$ for all code rates. Regular (3,6) mother code of rate of $R = 8/16$ with extension to $8/18, 8/20, 8/22$ and $8/24$. Regular mother code of rate of $R = 8/32$ punctured to get code rates $8/30, 8/28$ and $8/24$. Unaltered (original) regular code of rate $8/24$ for comparison.

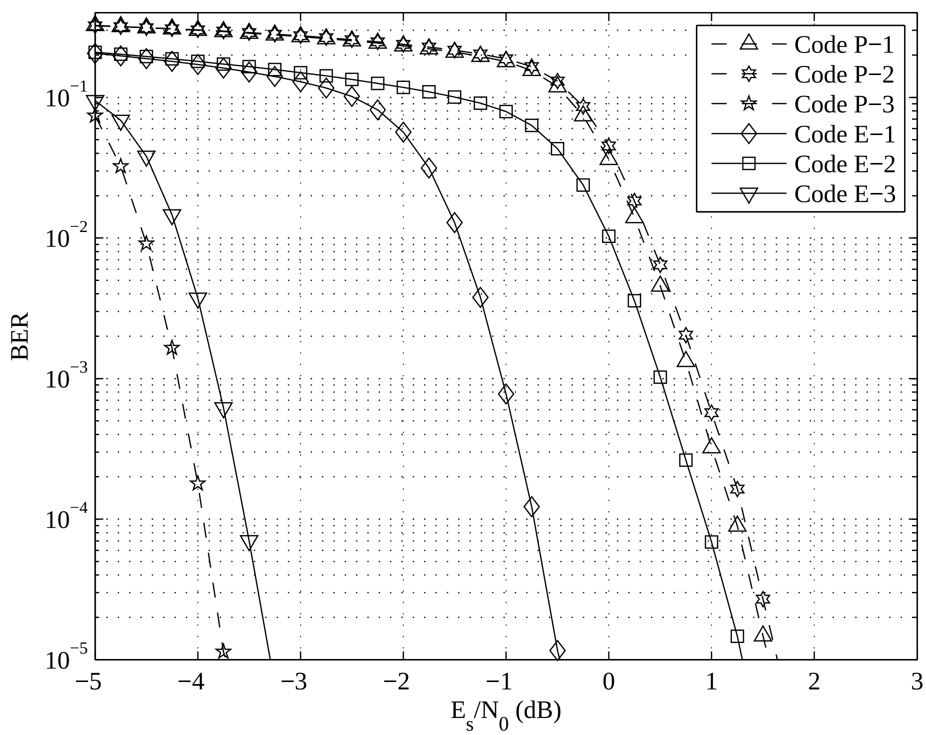


Figure 2.14: BER performance of punctured codes P-1, P-2, P-3 and extended codes E-1, E-2, E-3 over AWGN channel. Information length $k = 512$ for all codes. Codeword length is 1024 for codes P-1, P-2, E-1, E-2. Codeword length for P-3 is 2048. Codeword length for E-3 is 1536.

2.7 Conclusion

LDPC codes are capacity-achieving block codes. LDPC codes with good error correction capability are obtained by semi-random construction of **H** matrix. The BER performance of LDPC codes improves with larger block sizes. The decoding of LDPC codes almost converges with 100 decoder iterations of SPA.

Rate-compatible LDPC codes are constructed by puncturing and extension. Puncturing was done to construct high rate codes from a low rate mother code whereas extension method was used to construct low rate codes from a high rate mother code. The codes constructed by extension have better performance than punctured codes over AWGN channels.

We proposed a modification to the extended LDPC codes and compared it with punctured LDPC codes. The BER comparison of this modification to extended LDPC codes with punctured LDPC codes shows that the extended LDPC codes outperforms the punctured LDPC codes while keeping the codeword length same for both codes.

CHAPTER 3

COOPERATIVE DIVERSITY WITH LDPC CODES

3.1 Introduction

Cooperative diversity is a form of space diversity which exploits the space diversity by using a collection of distributed antennas belonging to different terminals. Earlier work on cooperative diversity was proposed in [19], [20]. Later on, efficient protocols and outage behavior was analyzed in [54], [63], [64]. An overview on cooperative diversity has been published in [21]. Figure 3.1 shows an illustration for cooperative diversity with three terminals. T_1 and T_2 can be two users or mobile stations whereas T_3 can be the destination or base station. For time-division systems, we assume half duplex operation for all terminals, i.e. no terminal can transmit or receive at the same time.

This chapter is organized as follows: In section 3.2, the system model for

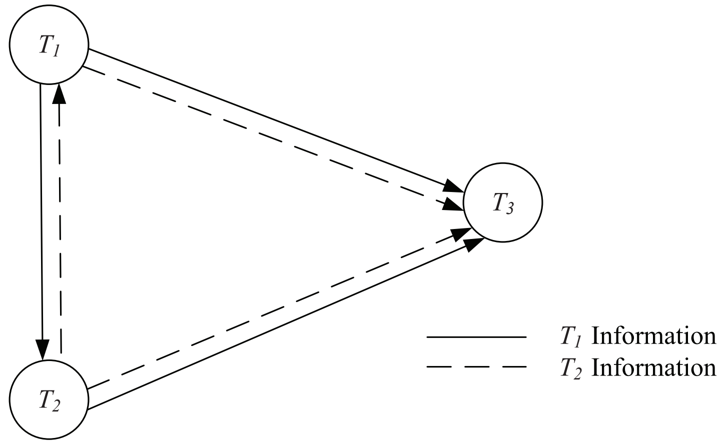


Figure 3.1: Cooperative diversity with three terminals.

the coded cooperative diversity will be discussed. Coded cooperative diversity with punctured LDPC codes is discussed in section 3.3.1. The modified extended codes proposed in section 2.5.3 are integrated in cooperative diversity scheme in 3.3.2. We compare the complexity for punctured and extended LDPC codes in cooperative diversity in section 3.3.3. The chapter is concluded with simulation results and discussion of LDPC coded cooperative diversity in slow fading (block fading) channels.

3.2 System Model for Coded Cooperative Diversity

We assume a time-division based (TDMA) system in which time slots are reserved for each transmitting terminal or user T_u where $u \in \{1, 2\}$. The same system model can be extended to any system with orthogonal channels. Each user encodes its data into a codeword N. The codeword N is divided into weaker codewords denoted by N_r where $r \in \{1, 2\}$. Each user requires two time slots to transmit the codeword N. The first time slot for each user is reserved for its own first codeword $N_1^{T_u}$. T_1 transmits in its first time slot acting as a source S. This codeword $N_1^{T_1}$ is received by the destination and the second user T_2. Similarly, user T_2 sends its own codeword $N_1^{T_2}$ in its first time slot. Each user and destination attempt to decode the transmission received and check its integrity by cyclic redundancy check (CRC). Users do not have the knowledge whether their transmission is received correctly or not. Therefore, they act independently in their second time slots. As a result, the transmission in the second time slot by each user can be divided into four cases. These cases are shown pictorially in Figure 3.2 and the time frame structure is shown in Table 3.1. In Case 1, both users successfully

Table 3.1: Transmission frame structure.

Case 1	T_1	$N_1^{T_1}$		$N_2^{T_2}$	
	T_2		$N_1^{T_2}$		$N_2^{T_1}$
Case 2	T_1	$N_1^{T_1}$		$N_2^{T_1}$	
	T_2		$N_1^{T_2}$		$N_2^{T_2}$
Case 3	T_1	$N_1^{T_1}$		$N_2^{T_1}$	
	T_2		$N_1^{T_2}$		$N_2^{T_1}$
Case 4	T_1	$N_1^{T_1}$		$N_2^{T_2}$	
	T_2		$N_1^{T_2}$		$N_2^{T_2}$

decode the codewords received from their partners. Each user will re-encode the data into the second codeword and send it to the destination. In this case both users are cooperating. In Case 2, both users fail to decode the transmission by

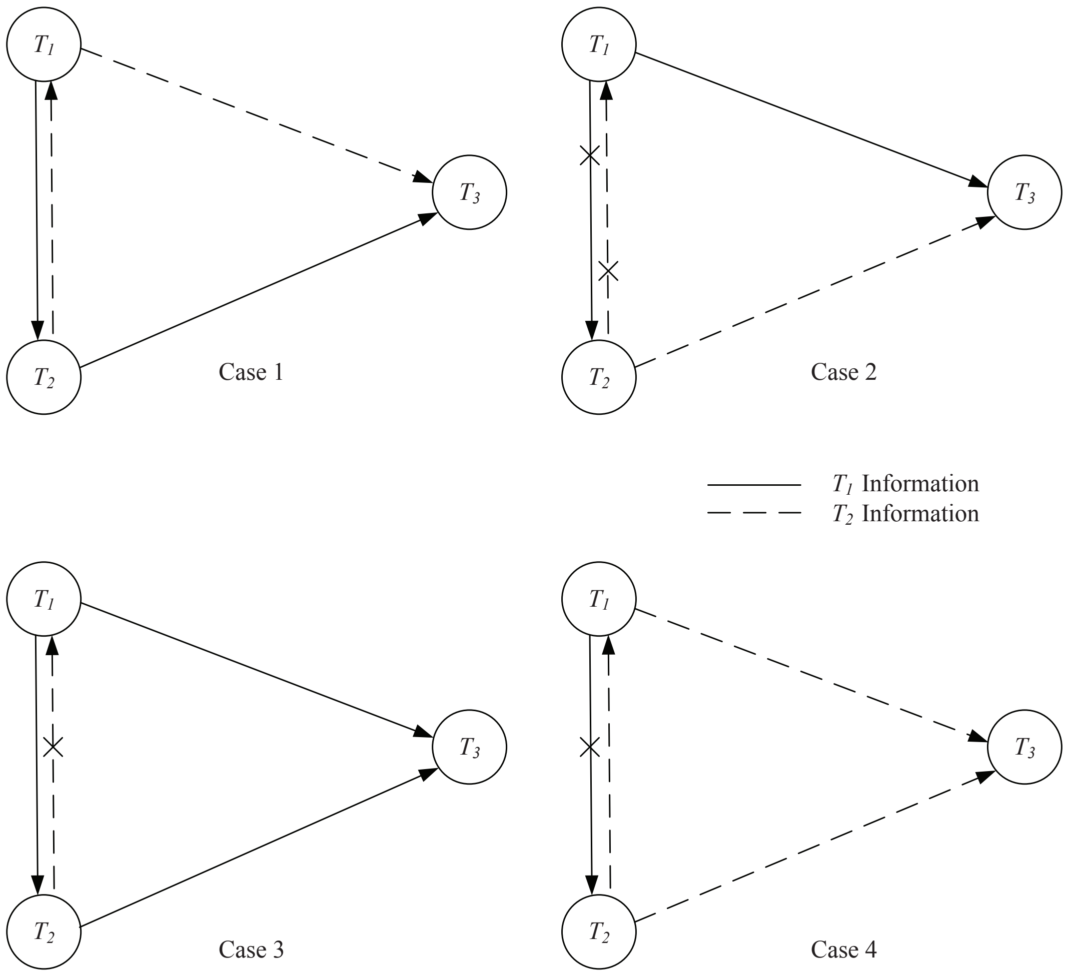


Figure 3.2: Four cooperative diversity cases based on transmission in second time slot of each user.

their partners. Therefore, they will continue to send there own second codewords leading to the no cooperation mode. In Case 3, T_1 fails to decode the transmission from T_2 while T_2 successfully decodes the transmission of T_1. In this case, both users will transmit the second codeword for T_1, i.e., $N_2^{T_1}$. In Case 4, T_2 fails to decode the transmission from T_1 while T_1 successfully decodes the transmission from T_2. In this case, both users will transmit the codeword $N_2^{T_2}$ for T_2.

The baseband-equivalent discrete-time system model for the source to the relay channel can be modeled as

$$y_R[\mathbf{n}] = \alpha_{S,R} \ x_S[\mathbf{n}] + \eta_R[\mathbf{n}] \tag{3.1}$$

whereas, the channel model from source to destination is given by

$$y_D[\mathbf{n}] = \alpha_{S,D} \ x_S[\mathbf{n}] + \eta_D[\mathbf{n}] \tag{3.2}$$

Assuming the total time equal to $\mathbb{N}$ for the two users to transmit two codewords (N_1 and N_2), the four time slots can be differentiated as follow:

- First odd time slot: $S = T_1$, $R = T_2$, $x_s = N_1^{T_1}$, $\mathbf{n} = 1, \ldots, \mathbb{N}/4$
- Second odd time slot: $S = T_1$, $R = T_2$, $x_s \in \{N_2^{T_1}, N_2^{T_2}\}$, $\mathbf{n} = \mathbb{N}/2 + 1, \ldots, 3\mathbb{N}/4$
- First even time slot: $S = T_2$, $R = T_1$, $x_s = N_1^{T_2}$, $\mathbf{n} = \mathbb{N}/4 + 1, \ldots, \mathbb{N}/2$
- Second even time slot: $S = T_2$, $R = T_1$, $x_s \in \{N_2^{T_1}, N_2^{T_2}\}$, $\mathbf{n} = 3\mathbb{N}/4 +$

$1, \ldots, \mathbb{N}$

with the destination $D = T_3$, $\mathbb{N}$ is the total duration of one frame for the transmission of two codewords by both users, the channel coefficients α are Rayleigh distributed and $\eta \sim \mathcal{N}(0, \sigma^2)$. The odd time slots are reserved for T_1 whereas the even time slots are reserved for T_2.

At the destination T_3, each user's first codeword is decoded, if CRC fails, then the second codeword is decoded. If both fails, then the two packets are combined and decoded jointly. We call this decoding process *three-step decoding.* In Case 3 and Case 4, the same codewords received are combined optimally by maximal ratio combiner (MRC). The level of cooperation is defined as the ratio of the codeword bits sent by the partner to the total number of bits in the codeword N_2/N.

We will assume the channel to be the block fading channel [65], [66] which is modeled as follows

$$y_{ij} = \alpha_i x_{ij} + \eta_{ij} \tag{3.3}$$

where $i = 1, 2, \ldots, M$ and $j = 1, 2, \ldots, h$ and each codeword is divided into M blocks and each block is h bits long. This is also a discrete-time channel model in which we have dropped the index $\mathbf{n}$ for simplicity. Therefore, the total length of the codeword is $M \times h$. The channel coefficients α_i are i.i.d with Rayleigh distribution for every block and $n_{ij} \sim \mathcal{N}(0, \sigma^2)$. For the special case of $M = 1$, the channel coefficient α remains constant for complete codeword.

We also assume that all terminals are perfectly synchronized with each other.

Moreover, all the channel estimates are perfectly known at the receiving terminals.

3.3 Cooperative Diversity with LDPC codes

In this section, the design of LDPC codes in cooperative diversity will be discussed. The punctured LDPC codes in cooperative diversity will be used for comparison with extended LDPC codes. We will integrate the extended LDPC codes in cooperative diversity with three-step decoding.

3.3.1 Cooperative Diversity with Punctured LDPC Codes

The LDPC codes are punctured to generate two weaker codewords. In the cooperative diversity framework, the codeword N is generated by an overall code rate k/n parity-check matrix and punctured periodically to generate two weaker codewords N_1 and N_2. Each of N_1 and N_2 can be decoded alone as a complete codeword. At the decoder, erasures are inserted at the punctured locations. At the destination, three-step decoding is applied. The codeword N_1 is decoded in the first step of decoding with erasures at the punctured locations. If CRC for the codeword fails, then N_2 is decoded with erasures at the punctured locations. If both N_1 and N_2 are irrecoverable, then N_1 and N_2 are concatenated and decoded jointly. The concatenation is similar to interleaving the two codewords together. This interleaving effect increases the diversity gain.

3.3.2 Cooperative Diversity with Extended LDPC Codes

We use the design of extended LDPC codes discussed in section 2.5.3 in the cooperative diversity framework to achieve three-step decoding at the receiver. (A part of this work has been published in [67].) The codeword N_1 is generated by using the generator matrix obtained from $\mathbf{H_1}$ of dimensions $m \times n$ leading to a high code rate $R = k/n$. Using Eq. 2.23, N_1 takes the following form

$$N_1 = [i \quad p_1]_{1\times n} \tag{3.4}$$

where i is the information part and p is the parity part in the codeword N_1. The second codeword N is generated by the generator matrix $\mathbf{G}_2$ mentioned in Eq. 2.24, in the following form

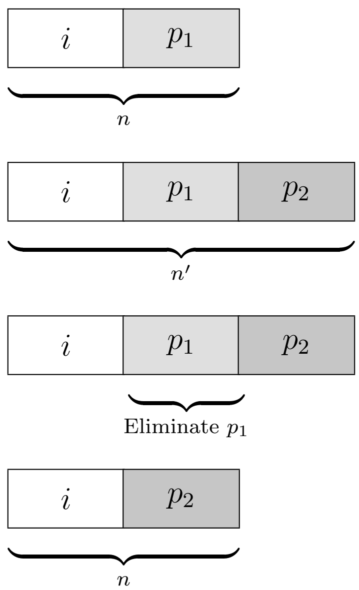
$$N = [i \quad p_1 \quad p_2]_{1\times n'} \tag{3.5}$$

where p_1 is the same parity part as in N_1 and p_2 is the extended parity part. This second codeword has a low code rate of $R' = k/n'$ where n' is the size of this extended codeword N. The codeword N_1 is embedded in codeword N. The codeword N is further modified to generate a codeword N_2 of length n. The second codeword is transmitted in the following format

$$N_2 = [i \quad p_2]_{1\times n} \tag{3.6}$$

which makes the length of the codeword N_2 equal to n. The generation of codewords N_1 and N_2 is summarized below.

Encoding



Codeword N_1 generated by $\mathbf{H}_1$

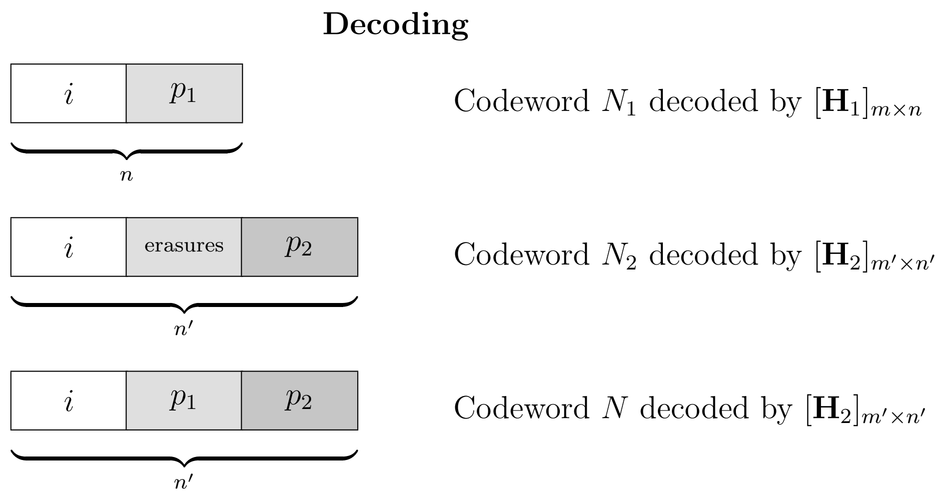
Codeword N_2' generated by $\mathbf{H}_2$

Eliminate p_1 from N_2'

Codeword N_2 generated from N_2'

At the receiver, in the first step N_1 is decoded using $\mathbf{H}_1$ of dimension $m \times n$. If the codeword is not successfully recovered in the first step, then the codeword is decoded using $\mathbf{H}_2$ matrix of size $m' \times n'$ with erasures inserted at p_1 of N_2. If the decoding fails in first two steps, then the codeword is concatenated and decoded on matrix $\mathbf{H}_2$ to recover the codeword N. The three-step decoding at the destination is summarized below.

The first step of decoding is done to decode the direct transmission from a user to destination. The second step of decoding is required to decode transmission from the relay to destination in cooperative behavior. Suppose, we have a large number of bits in error after the first step of decoding and there are no errors after the second step of decoding. If we skip step two and go directly to step three which is the joint decoding of both codewords, then we cannot guarantee



zero errors after decoding. The reason is that we cannot recover the information without errors because of the biased behavior of first codeword. That is why, we decode the second codeword with erasures which are unbiased values. Also, the decoding is done successively which means that if the destination receives the packet without errors at any step of decoding, then it skips the next steps. This is depicted in Figure 3.10.

3.3.3 Complexity Comparison

We assume an overall code rate of k/n for punctured codes as reference for complexity analysis. The punctured codes are decoded on $\mathbf{H}$ matrix of size $(n-k)\times n$. The approximate complexity in encoding for punctured and extended LDPC codes in cooperative diversity is presented in Table 3.2. The encoding complexity for extended codes is reduced by half for codeword N_1 as compared to punctured LDPC codes case. The decoding complexity using SPA for both the punctured and extended LDPC codes cases is mentioned in Table 3.3. The products and additions are found approximately for one iteration. The decoding is done in

Table 3.2: Comparison of encoding complexity between punctured and extended LDPC coded cooperative diversity.

		AND	XOR
Punctured LDPC	N_1	nk	$n(k-1)$
	N_2	nk	$n(k-1)$
Extended LDPC	N_1	$\frac{n}{2}k$	$\frac{n}{2}(k-1)$
	N_2	$\frac{3n}{4}k$	$\frac{3n}{4}(k-1)$

Table 3.3: Comparison of decoding complexity between punctured and extended LDPC coded cooperative diversity (approximated for one decoder iteration.)

		Products	Additions
Punctured LDPC	N_1	$(n-k)(w_r-2)$	nw_c
	N_2	$(n-k)(w_r-2)$	nw_c
	N	$(n-k)(w_r-2)$	nw_c
Extended LDPC	N_1	$(\frac{n}{2}-k)(w_r-2)$	$\frac{n}{2}w_c$
	N_2	$(\frac{3n}{4}-k)(w_r-2)$	$\frac{3n}{4}w_c$
	N	$(\frac{3n}{4}-k)(w_r-2)$	$\frac{3n}{4}w_c$

3 steps, therefore, all three cases for N_1, N_2 and N are shown. The decoding complexity remains the same for the punctured codes because the codewords are decoded on the same $\mathbf{H}$ matrix. For the extended codes, the decoding complexity for N_1 is reduced by almost half as compared to punctured codes because it is decoded on $\mathbf{H}_1$ whereas the decoding complexity for N_2 and combined codeword N is the same because N_2 is decoded with erasure insertion and both are decoded on $\mathbf{H}_2$ matrix of same size but still they have less complexity than punctured codes. Therefore, extended LDPC codes has less complexity as compared to punctured

LDPC codes. (A part of this work has been published in [68].)

3.4 Simulation Results and Discussion

All of these simulation results have been plotted as BER versus the channel SNR. The plots with BER versus information bit SNR will be identical with a shift of 10log$R$ dB. We assume very slow fading (block fading) channel in which the channel coefficient remains constant for the transmission of two time slots for each user. The BER for various values of inter-user channel SNR have been plotted. The level of cooperation is 50% in all simulation results.

Figures 3.3 and 3.4 show the BER and FER of punctured LDPC codes with cooperative diversity, respectively. The information bits were 512 and the length of the codewords N_1 and N_2 was 1024. The codewords were decoded on $\mathbf{H}$ matrix of size 1536×2048. The inter-user channel is the same in both directions, i.e., the channel coefficient from user 1 to user 2 α_{T_1,T_2} is equal to the channel coefficient from user 2 to user 1 α_{T_2,T_1}. The perfect inter-user channel shows the diversity gain achieved through coded cooperation. At a BER of 10^{-3}, a gain of 10 dB has been achieved with punctured LDPC codes with perfect inter-user channel versus poor inter-user channel. The performance curves for BER for 0 dB, 10 dB and 20 dB have also been plotted in the same figures. There is a significant improvement in BER with an increase in inter-user channel SNR.

The BER and FER performance curves for the extended LDPC coded cooperative diversity is shown in Figures 3.5 and 3.6, respectively. The information block size $k = 512$ is the same for both punctured and LDPC codes and the codewords N_1 and N_2 transmitted are also of same length equal to 1024 for fair comparison. The extended codes are decoded on $\mathbf{H}_1$ of size 512×1024 for the first step and $\mathbf{H}_2$ of size 1024×1536 for the next two steps of decoding. The gain is also 10 dB for the perfect inter-user channel as compared to the worse inter-user channel. The performance curves for BER for 0 dB, 10 dB and 20 dB have also been plotted in the same figures.

Figure 3.7 shows a comparison of punctured and extended LDPC codes for the 10 dB inter-user channel. The extended LDPC codes perform better at very low SNR as compared to punctured LDPC codes in cooperative diversity. However, the error rate for punctured LDPC codes reduces as compared to extended LDPC codes at higher SNR at the destination. The punctured LDPC codes performs better than extended LDPC codes in cooperative diversity by 1.7 dB at a BER of 10^{-3}. This result requires some explanation. The combined codeword is decoded on $\mathbf{H}_2$ of size 1024×1536 in extended codes whereas the combined codeword is decoded on $\mathbf{H}$ matrix of size 1536×2048. There is always a better chance of codeword recovery by using $\mathbf{H}$ matrix of larger size for decoding. Therefore, punctured LDPC codes performs better as compared to extended LDPC codes in cooperative diversity. However, (based on the discussion in section 3.3.3) punctured LDPC codes have higher encoding and decoding complexity as compared to extended LDPC codes. Therefore, there is a trade-off between error performance and encoding/decoding complexity between punctured and extended LDPC codes in cooperative diversity.

The results discussed so far has same inter-user channel. In Figure 3.8, BER performance has been shown for the extended LDPC codes for same inter-user channel as well as mutually independent channel. The channel coefficient for user 1 to user 2 is independent from the channel coefficient for user 2 to user 1, i.e., $\alpha_{T_1,T_2} \neq \alpha_{T_2,T_1}$. The BER performance curves show that the error rate is less for mutually independent inter-user channel. For the no cooperation scenario, the BER performance will be the same because only Case 2 (Table 3.1) is dominating. Similarly, for the perfect inter-user channel, the BER performance will also be the same because Case 1 (Table 3.1) is dominating. However, at a BER of 10^{-3} with 10 dB inter-user channel SNR, the BER performance with mutually independent inter-user channel is better than same or reciprocal inter-user channel by almost 1 dB. With the same inter-user channel, Case 1 and Case 2 are dominant (Table 3.1). However, with mutually independent inter-user channel, Case 3 and Case 4 are dominant (Table 3.1). In Case 3 and Case 4, there are three codewords for one user's data and these three codewords have unaltered information bits. By combining these three codes, a diversity gain is achieved which results in better performance with low error rate.

In Figure 3.9, BER versus user 2 SNR at base station has been plotted. The inter-user channel is mutually independent. The user 1 channel SNR at base station is constant at 5 dB, i.e., the channel between user 1 and destination is a static channel. The user 2 SNR at the base station is varying from 0 dB to 20 dB. This figure shows the BER for user 2. The error rate for user 2 reduces as the inter-user channel SNR improves. This result shows that there is significant reduction in error-rate for a particular user when its partner is in static condition which is the result of the gain achieved by cooperative diversity.

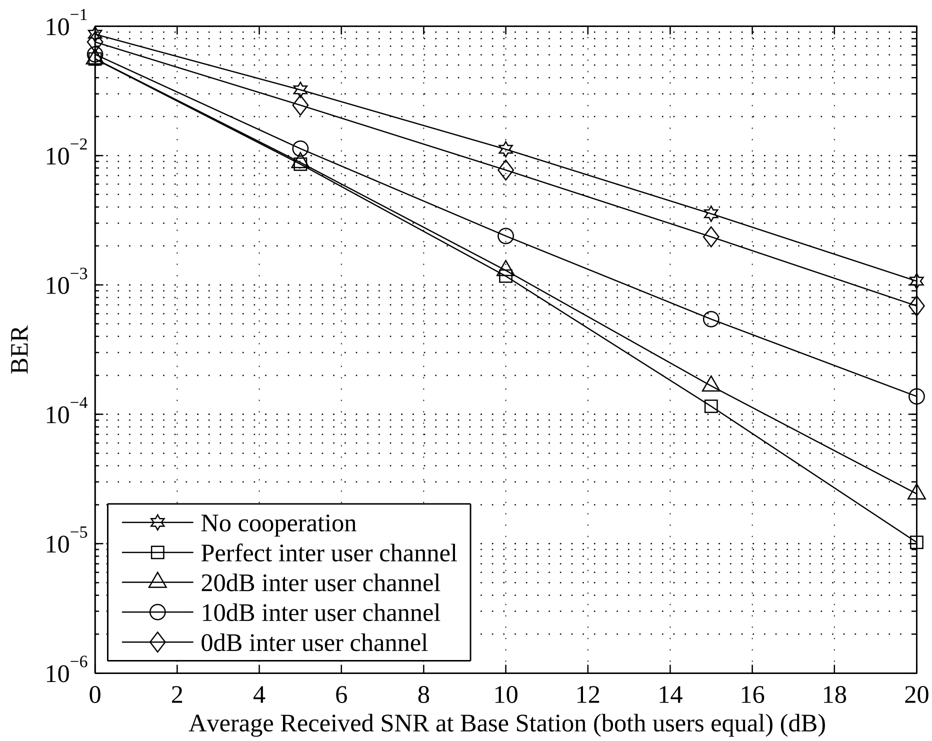


Figure 3.3: Cooperative diversity with punctured LDPC codes (BER). Information bits = 512, 50% cooperation, both users equal SNR, very slow fading (block fading) channel, same inter-user channel.

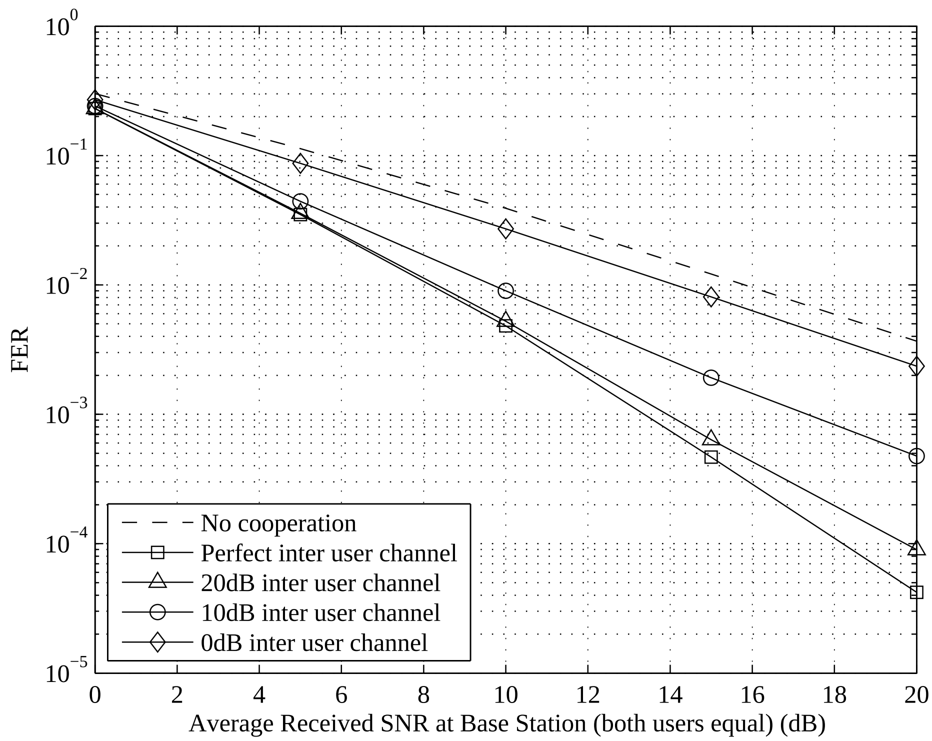


Figure 3.4: Cooperative diversity with punctured LDPC codes (FER). Information bits = 512, 50% cooperation, both users equal SNR, very slow fading (block fading) channel, same inter-user channel.

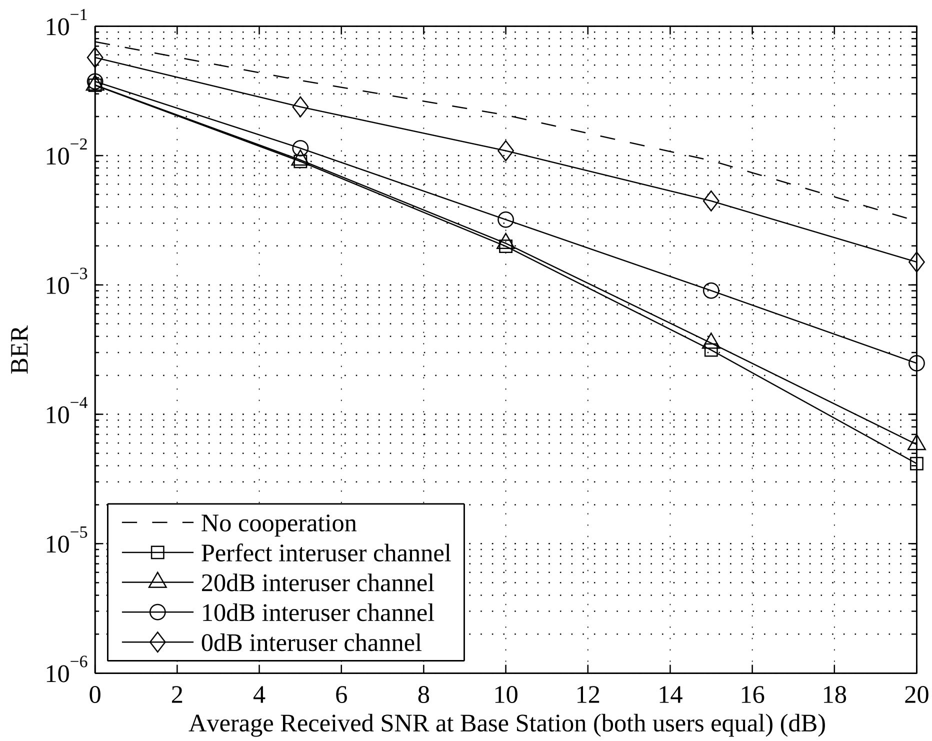


Figure 3.5: Cooperative diversity with extended LDPC codes (BER). Information bits = 512, 50% cooperation, both users equal SNR, very slow fading (block fading) channel, same inter-user channel.

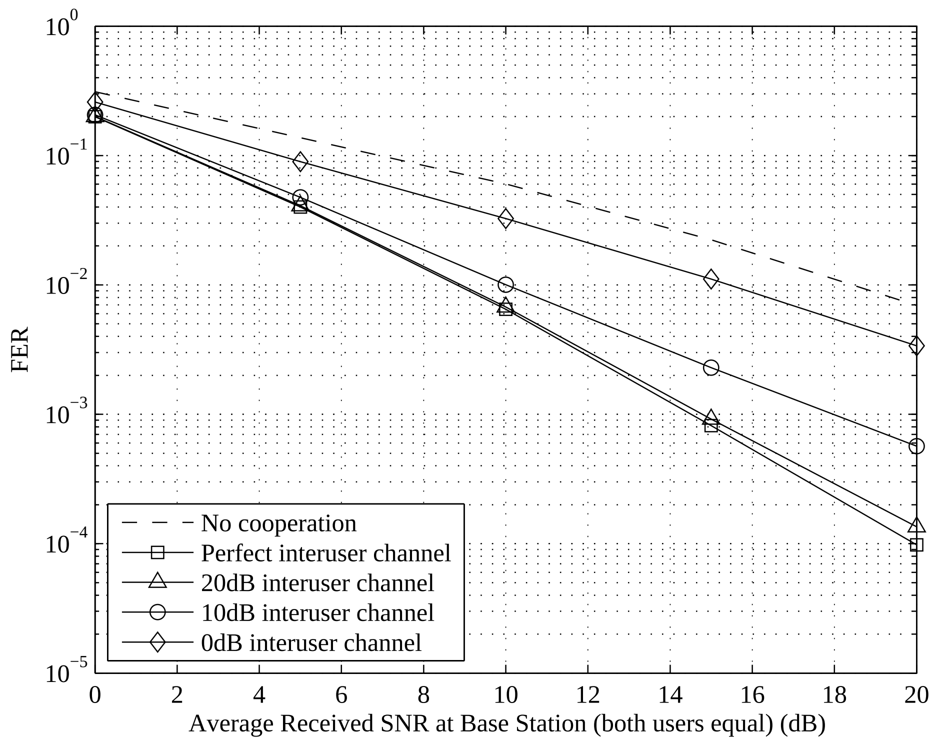


Figure 3.6: Cooperative diversity with extended LDPC codes (FER). Information bits = 512, 50% cooperation, both users equal SNR, very slow fading (block fading) channel, same inter-user channel.

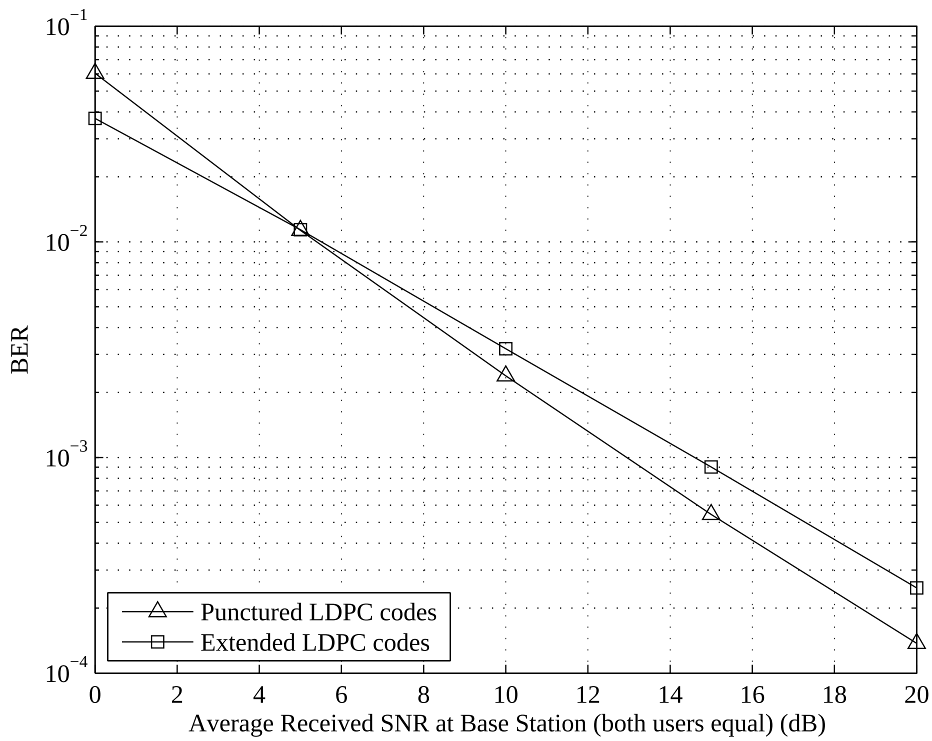


Figure 3.7: Cooperative diversity with punctured and extended LDPC codes (BER). Information bits = 512, 50% cooperation, both users equal SNR, very slow fading (block fading) channel, inter-user channel 10 dB, same inter-user channel.

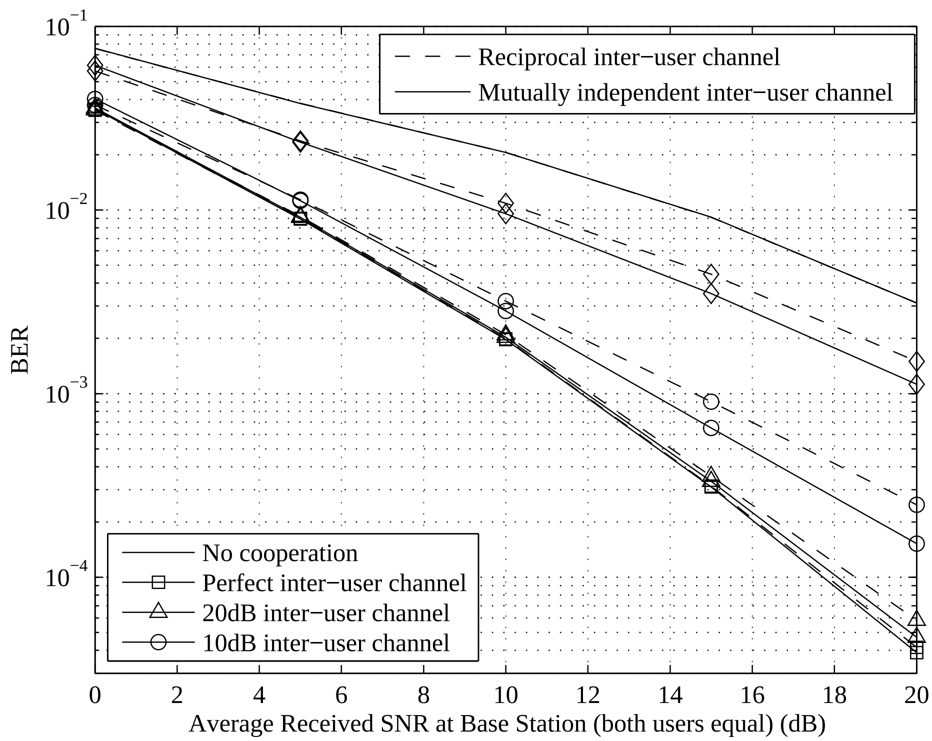


Figure 3.8: A bit error rate comparison with mutually independent channel and same inter-user channel in cooperative diversity with extended LDPC codes. Information bits = 512, 50% cooperation, both users equal SNR, very slow fading (block fading) channel.

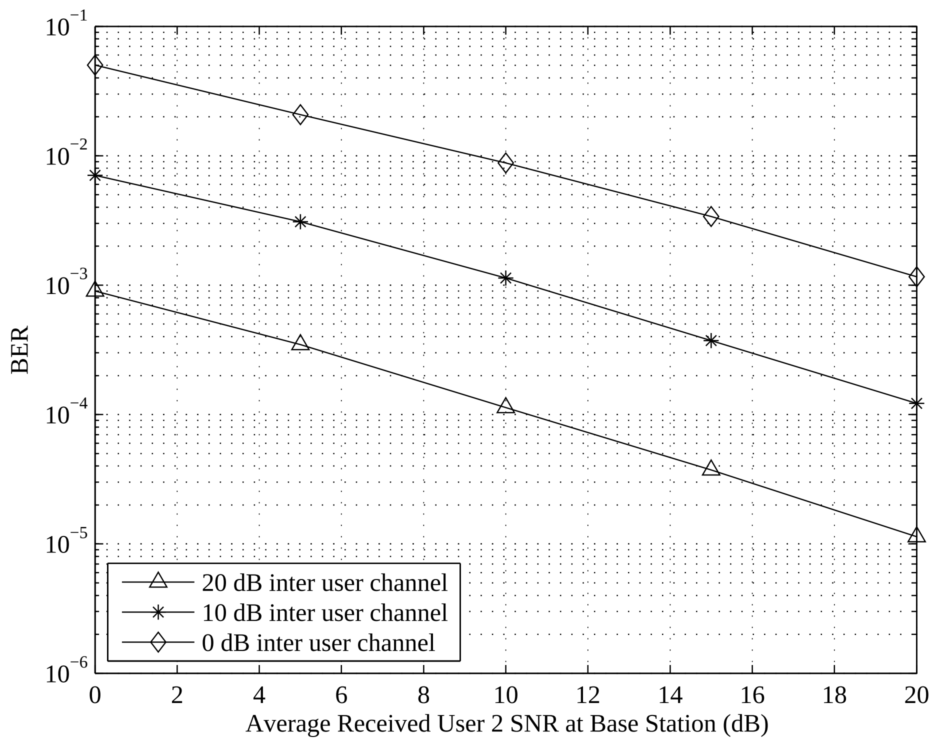


Figure 3.9: Bit error rate for user 2 in cooperative diversity with extended LDPC codes keeping the user 1 channel to destination static at 5 dB. Information bits = 512, 50% cooperation, user 2 SNR varying from 0 to 20 dB, very slow fading (block fading) channel.

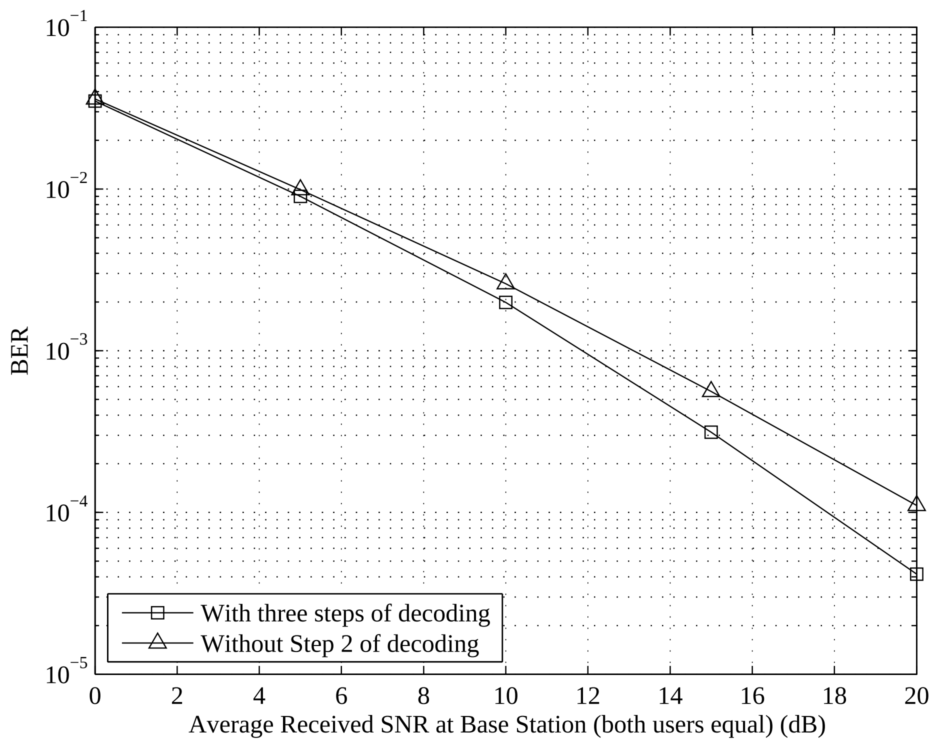


Figure 3.10: Effect of skipping the second step of decoding on BER. Cooperative diversity with extended LDPC codes (BER). Information bits = 512, 50% cooperation, both users equal SNR, very slow fading (block fading) channel, same and perfect inter-user channel.

3.5 Conclusion

We successfully integrated the proposed modification to the extended LDPC coded into cooperative diversity scheme. The decoding at the base station was successfully done in three steps. The overall performance of extended LDPC codes is close to punctured LDPC codes in cooperative diversity in terms of BER and FER over block fading channel. The punctured LDPC codes are slightly better in BER and FER performance than extended LDPC codes at higher average SNR at the base station. However, extended LDPC codes have much lower encoding/decoding complexity as compared to punctured LDPC codes in cooperative diversity scheme.

The extended LDPC codes performance is better with mutually independent inter-user channel as compared to same or reciprocal inter-user channel. Moreover, with one user to destination channel in static condition, the other user still performs significantly well over varying channel condition at the destination.

CHAPTER 4

FEEDBACK-BASED LDPC-CODED COOPERATIVE DIVERSITY

4.1 Introduction

The coded cooperative diversity schemes discussed in Chapter 3 have a constant throughput equal to the overall code rate. The throughput efficiency can be improved by exploiting the limited feedback from the destination. In this chapter, we will look into the throughput efficiency of extended LDPC codes. We propose two new ACK/NACK-based incremental relaying protocol for extended LDPC-coded cooperative diversity [69], [70].

This chapter is organized as follows: The first new incremental relaying protocol (Protocol I) will be introduced in section 4.2. The second protocol (Protocol

II) is discussed in section 4.3. The simulation results for both proposed protocols are discussed in section 4.5. Section 4.6 concludes the chapter.

4.2 Proposed Incremental Relaying Protocol I

We propose a novel incremental relaying protocol for LDPC-coded cooperative diversity. This protocol is a direct extension of the non-feedback-based protocol to the feedback-based protocol. This incremental relaying protocol is based on limited feedback from the destination. The feedback from the destination is an acknowledgement (ACK) if the codeword is received correctly at the destination or a negative-acknowledgement (NACK) if the codeword contains errors. The data integrity at the destination is verified by cyclic redundancy check (CRC). The protocol is designed according to time division based transmission without loss of generality. It can be extended into frequency division, code division or any other communication systems with orthogonal channels.

Automatic-repeat-request (ARQ) strategies are related to the source–destination retransmissions to ensure that the information is delivered to the destination correctly. The classical ARQ strategies can be extended to cooperative ARQ. In this work we will limit the discussion to the feedback provided for one frame transmission without loss of generality.

The coded cooperative diversity system model discussed in section 3.2 is a non-feedback-based system and will be used here for comparison purpose. The second codeword transmission by each user in the non-feedback-based system is based on the codewords received by each user in the previous time slot. As a result, there are four cases for the transmission in the second time slot by each user. If the transmission in the first time slot for a user is received correctly by the destination, then there is no need to send the second codeword in the second time slot for that user.

However, based on limited feedback from the destination, the transmission of extra codewords can be avoided which will enhance the overall throughput of the system. We will extend the same non-feedback-based system model (discussed in section 3.2) to feedback-based LDPC-coded cooperative diversity protocol. We assume 50% cooperation and extended LDPC-coded cooperative diversity discussed in section 3.3.2. We also assume that the feedback channel is protected with low rate channel coding and it is error free.

Consider the Case 1 of the non-feedback-based system in which user 1 successfully decodes transmission from user 2 and user 2 also successfully decodes transmission from user 1. The destination also attempts to decode the transmission in the first time slot from user 1 and user 2. There are further four cases based on the decoding at the destination. In the first case (we will call it Case 1.1), the codewords sent by both users are correctly decoded at the destination. The destination broadcasts the positive acknowledgement (ACK) to both users. Therefore, the second time slot for each user is available for new transmission and the throughput will be 50% rather 25% as it was in case of non-feedback-based system. In the second case (Case 1.2), the destination is unable to decode both users' first codewords. In this case, the destination broadcasts a negative acknowledgement (NACK) to both users. Each user sends the second codeword for its partner. In this case, the throughput is 25%. In the third case (Case 1.3), the codeword sent by user 1 is negatively acknowledged while the codeword sent by user 2 is positively acknowledged. The destination broadcasts the NACK of user 1 and ACK of user 2 respectively. There is no need to send the second codeword of user 2 and therefore leaving the time slot available for a new codeword. However, the second codeword for user 1 will be sent. Therefore, the throughput in this case is 33.3%. The fourth case (Case 1.4) is similar to the third one with the role of user 1 and user 2 interchanged. In this case, the throughput is also 33.3%. The complete time division based frame structure for Case 1 along with subcases is illustrated in Table 4.1.

The notation used to explain the feedback-based protocol is as follows:

T_1	:	User 1
T_2	:	User 2
T_3	:	Destination
$N_1^{T_1}$	:	First codeword sent by T_1
$N_2^{T_1}$	:	Second codeword sent by T_1
$N_1^{T_2}$	:	First codeword sent by T_2
$N_2^{T_2}$	:	Second codeword sent by T_2
ACK	:	Positive acknowledgement broadcasted by T_3
NACK	:	Negative acknowledgement broadcasted by T_3
$\bigoplus$	:	New codeword or new transmission
$\bigodot$	:	ACK/NACK
—	:	Transmission not allowed

Similar extension to feedback-based protocol have been made for the remaining three cases. The remaining cases are self explanatory and are illustrated in Table 4.2, 4.3 and 4.4.

Table 4.1: Feedback-based transmission frame structure (subcases for Case 1) with feedback from the destination.

Case 1.1	T_1	$N_1^{T_1}$	—	—	—	$\oplus$	—	—	—	50%
	T_2	—	—	$N_1^{T_2}$	—	—	—	$\oplus$	—	
	T_3	—	ACK	—	ACK	—	$\oplus$	—	$\oplus$	
Case 1.2	T_1	$N_1^{T_1}$	—	—	—	$N_2^{T_2}$	—	—	—	25%
	T_2	—	—	$N_1^{T_2}$	—	—	—	$N_2^{T_1}$	—	
	T_3	—	NACK	—	NACK	—	$\odot$	—	$\odot$	
Case 1.3	T_1	$N_1^{T_1}$	—	—	—	$\oplus$	—	—	—	33.3%
	T_2	—	—	$N_1^{T_2}$	—	—	—	$N_2^{T_1}$	—	
	T_3	—	NACK	—	ACK	—	$\oplus$	—	$\odot$	
Case 1.4	T_1	$N_1^{T_1}$	—	—	—	$N_2^{T_2}$	—	—	—	33.3%
	T_2	—	—	$N_1^{T_2}$	—	—	—	$\oplus$	—	
	T_3	—	ACK	—	NACK	—	$\odot$	—	$\oplus$	

Table 4.2: Feedback-based transmission frame structure (subcases for Case 2) with feedback from the destination.

Case 2.1	T_1	$N_1^{T_1}$	—	—	—	$\oplus$	—	—	—	50%
	T_2	—	—	$N_1^{T_2}$	—	—	—	$\oplus$	—	
	T_3	—	ACK	—	ACK	—	$\oplus$	—	$\oplus$	
Case 2.2	T_1	$N_1^{T_1}$	—	—	—	$N_2^{T_1}$	—	—	—	25%
	T_2	—	—	$N_1^{T_2}$	—	—	—	$N_2^{T_2}$	—	
	T_3	—	NACK	—	NACK	—	$\odot$	—	$\odot$	
Case 2.3	T_1	$N_1^{T_1}$	—	—	—	$\oplus$	—	—	—	33.3%
	T_2	—	—	$N_1^{T_2}$	—	—	—	$N_2^{T_2}$	—	
	T_3	—	ACK	—	NACK	—	$\oplus$	—	$\odot$	
Case 2.4	T_1	$N_1^{T_1}$	—	—	—	$N_2^{T_1}$	—	—	—	33.3%
	T_2	—	—	$N_1^{T_2}$	—	—	—	$\oplus$	—	
	T_3	—	NACK	—	ACK	—	$\odot$	—	$\oplus$	

Table 4.3: Feedback-based transmission frame structure (subcases for Case 3) with feedback from the destination.

Case 3.1	T_1	$N_1^{T_1}$	—	—	—	$\bigoplus$	—	—	—	50%
	T_2	—	—	$N_1^{T_2}$	—	—	—	$\bigoplus$	—	
	T_3	—	ACK	—	ACK	—	$\bigoplus$	—	$\bigoplus$	
Case 3.2	T_1	$N_1^{T_1}$	—	—	—	$N_2^{T_1}$	—	—	—	25%
	T_2	—	—	$N_1^{T_2}$	—	—	—	$N_2^{T_1}$	—	
	T_3	—	NACK	—	NACK	—	$\bigodot$	—	$\bigodot$	
Case 3.3	T_1	$N_1^{T_1}$	—	—	—	$N_2^{T_1}$	—	—	—	25%
	T_2	—	—	$N_1^{T_2}$	—	—	—	$N_2^{T_1}$	—	
	T_3	—	NACK	—	ACK	—	$\bigodot$	—	$\bigodot$	
Case 3.4	T_1	$N_1^{T_1}$	—	—	—	$\bigoplus$	—	—	—	50%
	T_2	—	—	$N_1^{T_2}$	—	—	—	$\bigoplus$	—	
	T_3	—	ACK	—	NACK	—	$\bigoplus$	—	$\bigoplus$	

Table 4.4: Feedback-based transmission frame structure (subcases for Case 4) with feedback from the destination.

Case 4.1	T_1	$N_1^{T_1}$	—	—	—	$\bigoplus$	—	—	—	50%
	T_2	—	—	$N_1^{T_2}$	—	—	—	$\bigoplus$	—	
	T_3	—	ACK	—	ACK	—	$\bigoplus$	—	$\bigoplus$	
Case 4.2	T_1	$N_1^{T_1}$	—	—	—	$N_2^{T_2}$	—	—	—	25%
	T_2	—	—	$N_1^{T_2}$	—	—	—	$N_2^{T_2}$	—	
	T_3	—	NACK	—	NACK	—	$\bigodot$	—	$\bigodot$	
Case 4.3	T_1	$N_1^{T_1}$	—	—	—	$\bigoplus$	—	—	—	50%
	T_2	—	—	$N_1^{T_2}$	—	—	—	$\bigoplus$	—	
	T_3	—	NACK	—	ACK	—	$\bigoplus$	—	$\bigoplus$	
Case 4.4	T_1	$N_1^{T_1}$	—	—	—	$N_2^{T_2}$	—	—	—	25%
	T_2	—	—	$N_1^{T_2}$	—	—	—	$N_2^{T_2}$	—	
	T_3	—	ACK	—	NACK	—	$\bigodot$	—	$\bigodot$	

4.3 Proposed Incremental Relaying Protocol II

We propose a second incremental protocol [71]. This protocol is also based on limited feedback from the destination. In Protocol I, the decision of transmission in the second time slot was taken in two steps. In the first step, the protocol decides the Case based on the inter-user transmission. In the second step, the ACK/NACK received from the destination is analyzed and decision is made whether the decided packet is required at the destination or not. In Protocol II, the decision for the second time slot transmission is made by jointly analyzing the inter-user transmission and ACK/NACK received from the destination for the first time slot.

We will use the following notation to explain the Protocol II.

$u_{1,D}^{(1)}$	:	Transmission from user 1 to destination D in time slot 1
$u_{2,D}^{(1)}$	:	Transmission from user 2 to destination D in time slot 1
$u_{1,2}^{(1)}$	:	Transmission from user 1 to user 2 in time slot 1
$u_{2,1}^{(1)}$	:	Transmission from user 2 to user 1 in time slot 1
$u_{1,D}^{(2)}$	:	Transmission from user 1 to user 2 in time slot 2
$u_{2,D}^{(2)}$	:	Transmission from user 2 to user 1 in time slot 2
d	:	Don't care
c	:	CRC verified or the transmission received correctly
e	:	CRC failed or the transmission received with errors
new	:	New transmission

The system model used is the same as discussed in section 3.2. In the first time slot, each user broadcasts the first codeword N_1 which is received by the respective partner and destination. The destination decodes $N_1^{T_1}$ and $N_1^{T_2}$ and sends ACK/NACK to the users in the feedback channel. Each user analyzes the partner transmission and ACK/NACK received by the destination and will make a decision for transmission in second time slot. In this protocol, we will have 6 Cases which are illustrated in Table 4.5. The detailed time-domain frame structure with feedback from destination is shown in Table 4.6.

Table 4.5: Cases for incremental relaying Protocol II

Case	$u_{1,D}^{(1)}$	$u_{2,D}^{(1)}$	$u_{1,2}^{(1)}$	$u_{2,1}^{(1)}$	$u_{1,D}^{(2)}$	$u_{2,D}^{(2)}$	Throughput
1	ACK	ACK	d	d	new	new	50%
2	NACK	ACK	c	d	$N_2^{T_1}$	$N_2^{T_1}$	25%
3	NACK	ACK	e	d	$N_2^{T_1}$	new	33.3%
4	ACK	NACK	d	c	$N_2^{T_2}$	$N_2^{T_2}$	25%
5	ACK	NACK	d	e	new	$N_2^{T_2}$	33.3%
6	NACK	NACK	d	d	$N_2^{T_1}$	$N_2^{T_2}$	25%

The description of each Case is as follows:

- Case 1: Both users transmission in the first time slot is successfully decoded at the destination. Therefore, the second time slot is available for a new transmission for both users.

- Case 2: Destination fails to decode $N_1^{T_1}$ but successfully decodes $N_1^{T_2}$. User 2 successfully decodes $N_1^{T_1}$. Therefore, in the second time slot, user 1 will transmit the second codeword $N_2^{T_1}$ for his own data whereas user 2 will also transmit $N_2^{T_1}$ for his partner (user 1).

- Case 3: Destination fails to decode $N_1^{T_1}$ but successfully decodes $N_1^{T_2}$. User 2 fails to decode $N_1^{T_1}$. Therefore, in the second time time slot, user 1 will transmit $N_2^{T_1}$ for his own data whereas user 2 will be sending new data.

- Case 4: Destination successfully decodes $N_1^{T_1}$ but fails to decode $N_2^{T_2}$. This is similar to Case 2 with the role of both users reversed.

- Case 5: Destination successfully decodes $N_1^{T_1}$ but fails to decode $N_2^{T_2}$. This is similar to Case 3 with the role of both users reversed.

- Case 6: Destination fails to decode both $N_1^{T_1}$ and $N_1^{T_2}$. Therefore, user 1 will send $N_2^{T_1}$ and user 2 will send $N_2^{T_2}$ for their own data.

4.4 Comments on Protocol I and Protocol II

In both protocols, the destination must know which Case has occurred to apply the decoding correctly. This can be achieved by a small overhead of flag bits. Protocol I has 16 cases which means it requires a 4-bit flag. Protocol II has 6 cases, therefore, a 3-bit flag is enough to indicate which Case has occurred.

The key difference between both protocols is the transmission in the second time slot. In Protocol I, the codeword to be transmitted in the second time slot is chosen based on inter-user reception in the first time slot. Once this decision has been taken, the feedback from the destination is analyzed whether the second codeword transmission is required or not. Consider Case 3.2 (Table 4.3) of Protocol I as an example. Both users are negatively acknowledged by the destination

Table 4.6: Feedback-based transmission frame structure for Protocol II with feedback from the destination.

Case 1	T_1	$N_1^{T_1}$	—	—	—	$\oplus$	—	—	—	50%
	T_2	—	—	$N_1^{T_2}$	—	—	—	$\oplus$	—	
	T_3	—	ACK	—	ACK	—	$\odot$	—	$\odot$	
Case 2	T_1	$N_1^{T_1}$	—	—	—	$N_2^{T_1}$	—	—	—	25%
	T_2	—	—	$N_1^{T_2}$	—	—	—	$N_2^{T_1}$	—	
	T_3	—	NACK	—	ACK	—	$\odot$	—	$\odot$	
Case 3	T_1	$N_1^{T_1}$	—	—	—	$N_2^{T_1}$	—	—	—	33.3%
	T_2	—	—	$N_1^{T_2}$	—	—	—	$\oplus$	—	
	T_3	—	NACK	—	ACK	—	$\odot$	—	$\odot$	
Case 4	T_1	$N_1^{T_1}$	—	—	—	$N_2^{T_2}$	—	—	—	25%
	T_2	—	—	$N_1^{T_2}$	—	—	—	$N_2^{T_2}$	—	
	T_3	—	ACK	—	NACK	—	$\odot$	—	$\odot$	
Case 5	T_1	$N_1^{T_1}$	—	—	—	$\oplus$	—	—	—	33.3%
	T_2	—	—	$N_1^{T_2}$	—	—	—	$N_2^{T_2}$	—	
	T_3	—	ACK	—	NACK	—	$\odot$	—	$\odot$	
Case 6	T_1	$N_1^{T_1}$	—	—	—	$N_2^{T_1}$	—	—	—	25%
	T_2	—	—	$N_1^{T_2}$	—	—	—	$N_2^{T_2}$	—	
	T_3	—	NACK	—	NACK	—	$\odot$	—	$\odot$	

in the first time slot and $N_1^{T_1}$ was received correctly by T_2. The decision is made that the next codeword that will be transmitted is $N_2^{T_1}$ which means priority is given to T_1 even if T_2 was negatively acknowledged.

Behavior of Protocol II differs from Protocol I in the second time slot transmission. In the second time slot transmission, the priority will be given to each user's own data. If a user is negatively acknowledged by the destination then irrespective of whatever it has received from his partner, it will be transmitting its own second codeword. These differences will become more evident and are discussed further in section 4.5.

4.5 Simulation Results and Discussion

We assume 50% cooperation and extended LDPC-coded cooperative diversity for both protocols. The inter-user channel is assumed to be mutually independent. Next, we discuss the simulations results of Protocol I and II respectively.

4.5.1 Protocol I

We will discuss the simulation results for Protocol I. Figure 4.1 shows the average throughput of the feedback-based protocol. The average throughput for non-feedback-based protocol is also shown for comparison. The throughput for the non-feedback system remains constant at 25%. At 20 dB SNR at the base station, the throughput for feedback-based protocol is 25% higher than the non-feedback-based protocol.

Figure 4.2 shows the average throughput with different inter-user channel SNRs. It is clear that there is no effect of inter-user channel SNR on the average throughput of the feedback-based protocol. Since, we are only exploiting the limited feedback from the destination, therefore, the average throughput is only effected by each user to the destination channel. As an example, consider Case 1.3 (perfect inter-user channel) and Case 2.3 (no cooperation). Both of these Cases have an overall throughput of 33.3%.

We try to find out the percentage occurrence of each type of transmission. The transmission from each user is categorized into three categories: own parity, partner parity and new transmission. In Figure 4.3, percentage occurrence of transmission from user 1 versus average received SNR at the base station has been plotted. Since, both users have equal average SNR at the base station, therefore, these graphs will be the same for user 2 as well. Figure 4.3 is the case in which inter-user channel is bad (no cooperation) and both users are not cooperating. As a result, user 1 will not be transmitting any codeword for its partner (user 2) and the partner parity is zero. At very low average SNR at the base station, user 1 will be sending its own second codeword (own parity) in its second time slot based on the feedback from the destination. As the average SNR at he base station improves, there is no need for second codeword for user 1. As a result, new transmission increases because of the feedback from the destination.

Figure 4.4 shows the percentage occurrence of each type of transmission for user 1 with 0 dB inter-user channel. As the inter-user channel SNR improves, both users start to cooperate and therefore increasing the transmission of partner parity. Similarly, the partner parity further improves with increase in inter-user channel SNR as shown in Figure 4.5 in which the inter-user channel SNR is 10 dB. With perfect inter-user channel (Figure 4.6), each user is sending for its partner most of the time and its own parity is almost zero, i.e., both users are fully cooperating. The new transmission by each user is dependent on the channel condition between user to the destination and the feedback from the destination.

Therefore, inter-user channel SNR do not effect the new transmission by each user.

Since, this protocol is a direct extension of non-feedback based system model in Chapter 3, therefore, the FER performance of Protocol I will be the same as presented in Figure 3.6 in Chapter 3.

4.5.2 Protocol II

Next we will discuss the simulation results of Protocol II. Figure 4.7 shows the average throughput of Protocol II. The best throughput of the protocol is achieved when there is no cooperation (worse inter-user channel) between the two users. As the inter-user channel improves, there will be more codewords for the respective partners to send (better cooperation) which will effectively reduce the overall throughput of the protocol.

Figure 4.8 shows the percentage occurrence of each type of transmission by user 1. Simulation results for user 2 will be the same as for user 1. This result is plotted for the worse inter-user channel with no cooperation between the users. Since, both users are not cooperating, therefore, they will be either sending their own parity or new codewords of their own.

Figure 4.9 is plotted with 10 dB inter-user channel SNR. As the inter-user channel improves, we can see that user 1 starts sending the parity (second codeword) for the partner (user 2) when the user 2 to destination channel is poor.

Figure 4.10 is plotted for perfect inter-user channel (both users fully cooperating). As the user 2 to destination channel improves, the need for the second codeword for user 2 reduces (based on feedback from destination). As a result, there is no need to send second codeword from user 1. That is why the partner parity plot decays as the destination SNR improves. The new transmission will be at maximum with good user to destination channel.

In Protocol II, the second time slot transmission has been changed as compared to Protocol I. Therefore, there will be a difference in the FER of protocol II. Figure 4.11 shows the FER of Protocol II in comparison with non-feedback-based system. Performance of Protocol II is slightly better than protocol I and non-feedback-based system. This slight improvement is the result of more cooperation between the two users at lower destination SNR. At higher SNR at destination, however, both protocols will perform almost the same.

Lastly, we compare the throughput of Protocol I and Protocol II in Figure 4.12. We mentioned in previous section that Protocol I will not have any effect on the throughput due to inter-user channel condition. Protocol II has effect of inter-user channel on the throughput because the decision of second time slot transmission is taken jointly on inter-user channel reception and feedback from the destination. However, Protocol II performs the same as Protocol I when there is no cooperation between the users which is the result as expected.

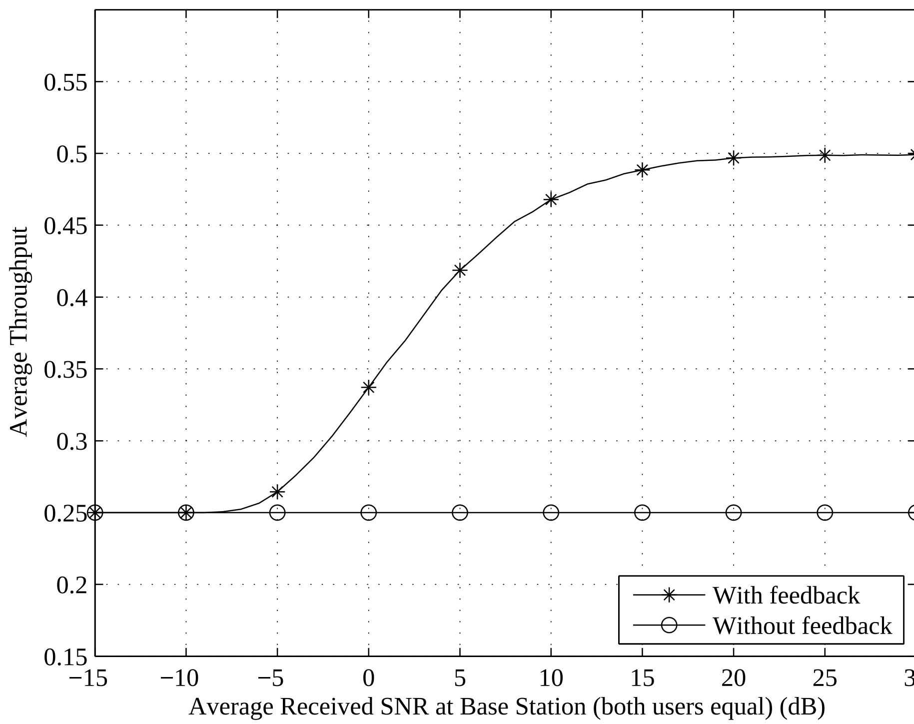


Figure 4.1: Protocol I – Average throughput for the ACK/NACK-based extended LDPC-coded cooperative diversity protocol. Information bits = 512, 50% cooperation, both users equal SNR, very slow fading (block fading) channel and mutually independent inter-user channel with average SNR equal to 10 dB.

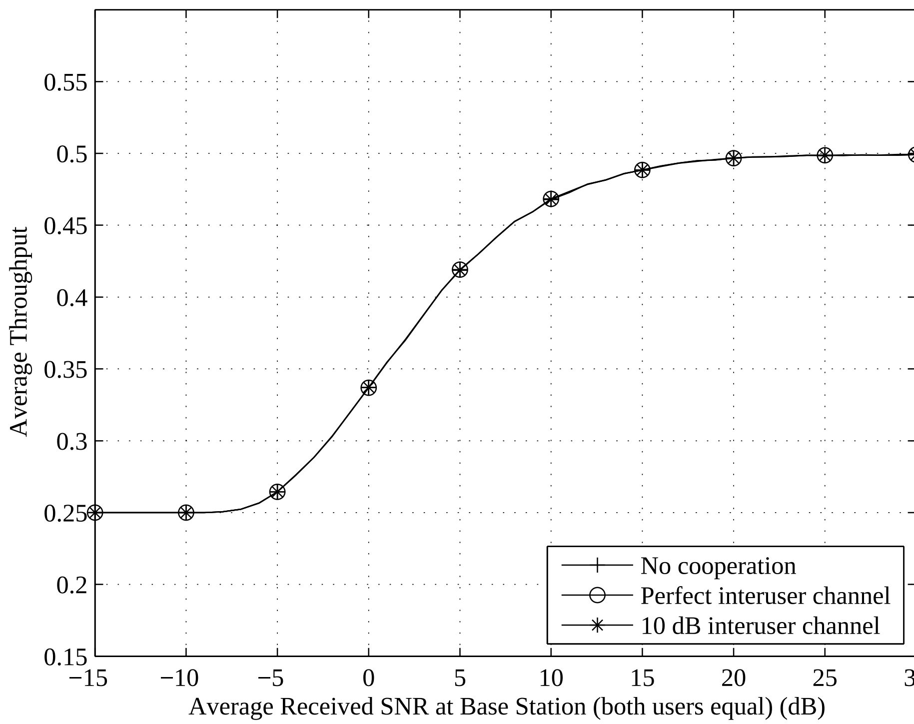


Figure 4.2: Protocol I – Average throughput for the ACK/NACK-based extended LDPC-coded cooperative diversity protocol. Information bits = 512, 50% cooperation, both users equal SNR, very slow fading (block fading) channel and mutually independent inter-user channel.

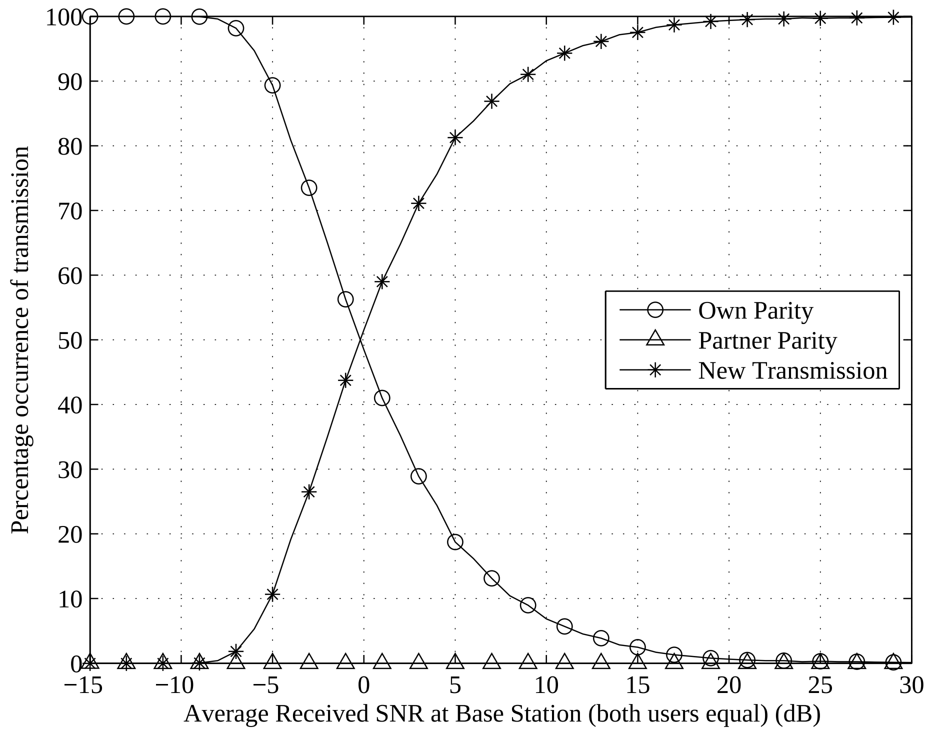


Figure 4.3: Protocol I – Percentage occurrence of each type of transmission (own parity, partner parity and new transmission) by user 1 for ACK/NACK based coded cooperative diversity with extended LDPC codes. Information bits = 512, 50% cooperation, both users equal SNR, very slow fading (block fading) channel, mutually independent inter-user channel and worse inter-user channel condition (no cooperation).

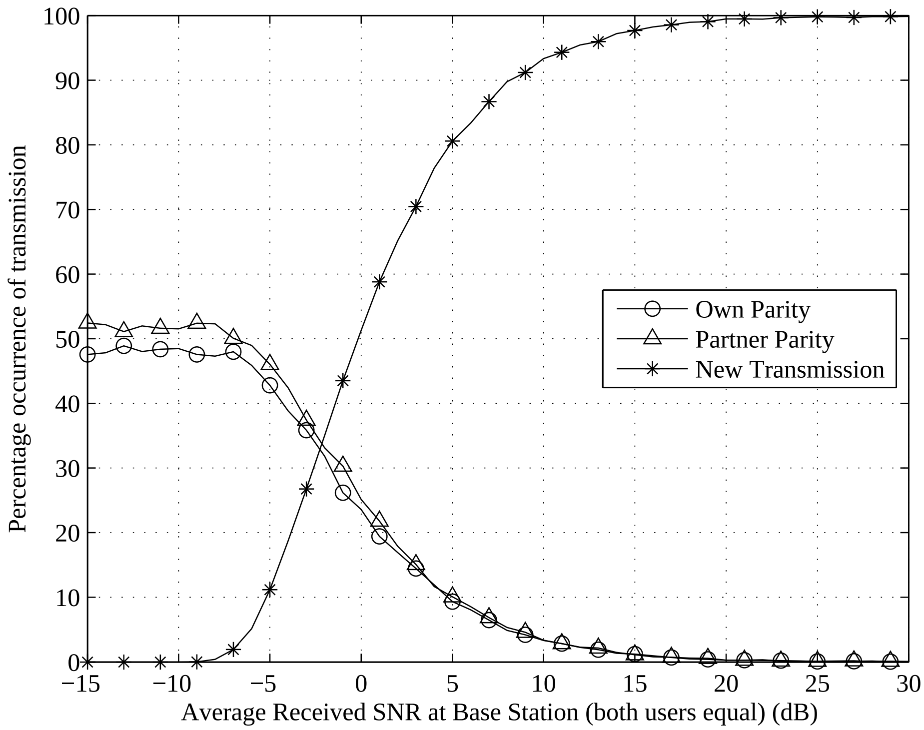


Figure 4.4: Protocol I – Percentage occurrence of each type of transmission (own parity, partner parity and new transmission) by user 1 for ACK/NACK based coded cooperative diversity with extended LDPC codes. Information bits = 512, 50% cooperation, both users equal SNR, very slow fading (block fading) channel and mutually independent inter-user channel with average SNR of 0 dB.

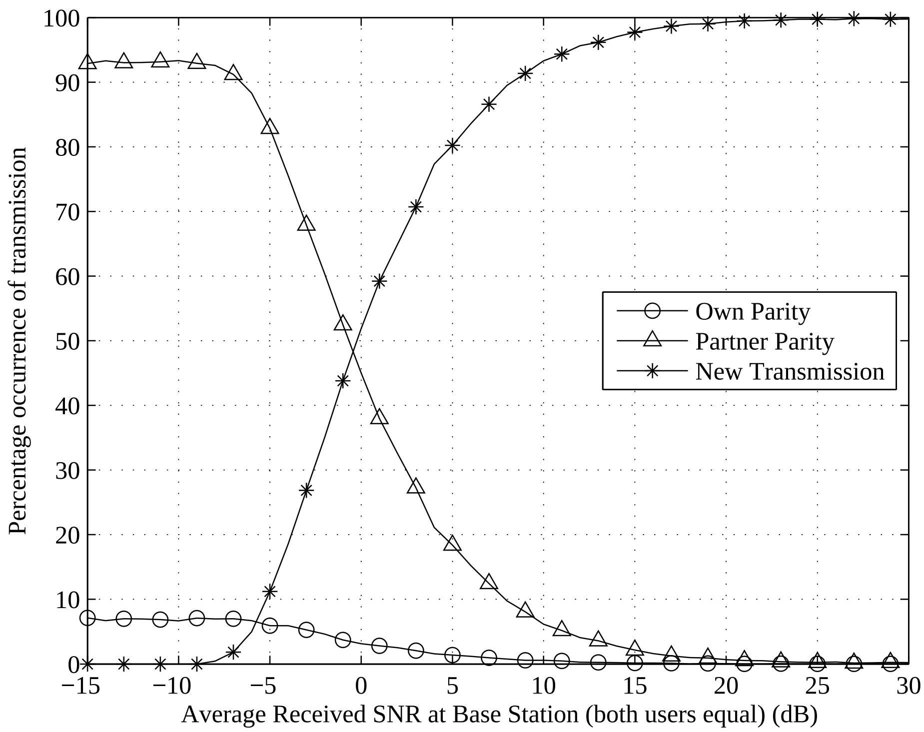


Figure 4.5: Protocol I – Percentage occurrence of each type of transmission (own parity, partner parity and new transmission) by user 1 for ACK/NACK based coded cooperative diversity with extended LDPC codes. Information bits = 512, 50% cooperation, both users equal SNR, very slow fading (block fading) channel and mutually independent inter-user channel with average SNR of 10 dB.

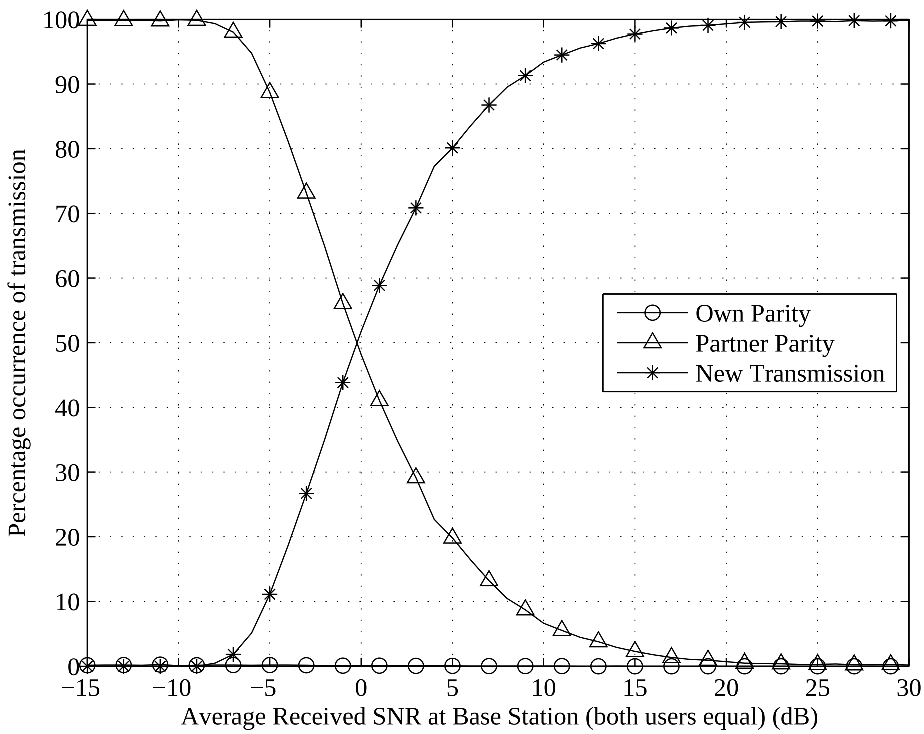


Figure 4.6: Protocol I – Percentage occurrence of each type of transmission (own parity, partner parity and new transmission) by user 1 for ACK/NACK based coded cooperative diversity with extended LDPC codes. Information bits = 512, 50% cooperation, both users equal SNR, very slow fading (block fading) channel, mutually independent inter-user channel and perfect inter-user channel condition.

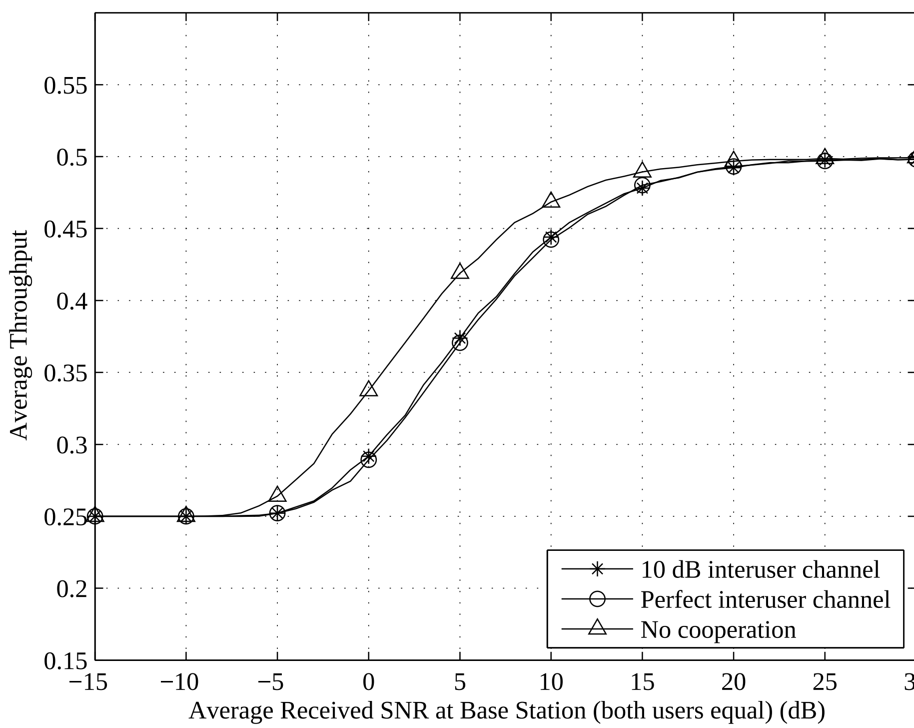


Figure 4.7: Protocol II – Average throughput for the ACK/NACK-based extended LDPC-coded cooperative diversity protocol. Information bits = 512, 50% cooperation, both users equal SNR, very slow fading (block fading) channel and mutually independent inter-user channel.

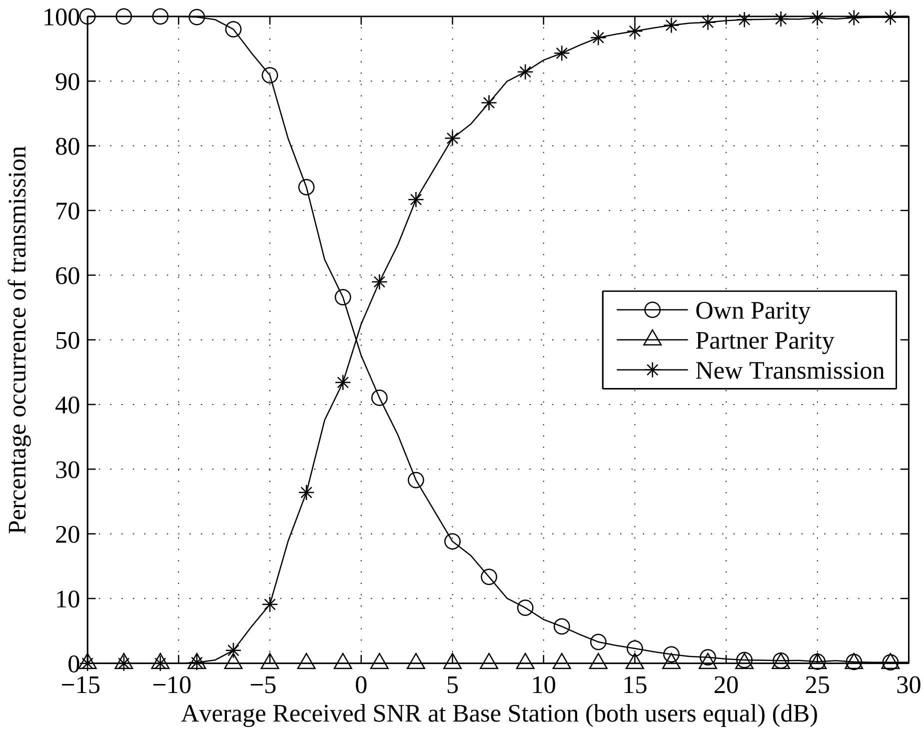


Figure 4.8: Protocol II – Percentage occurrence of each type of transmission (own parity, partner parity and new transmission) by user 1 for ACK/NACK based coded cooperative diversity with extended LDPC codes. Information bits = 512, 50% cooperation, both users equal SNR, very slow fading (block fading) channel, mutually independent inter-user channel and worse inter-user channel condition (no cooperation).

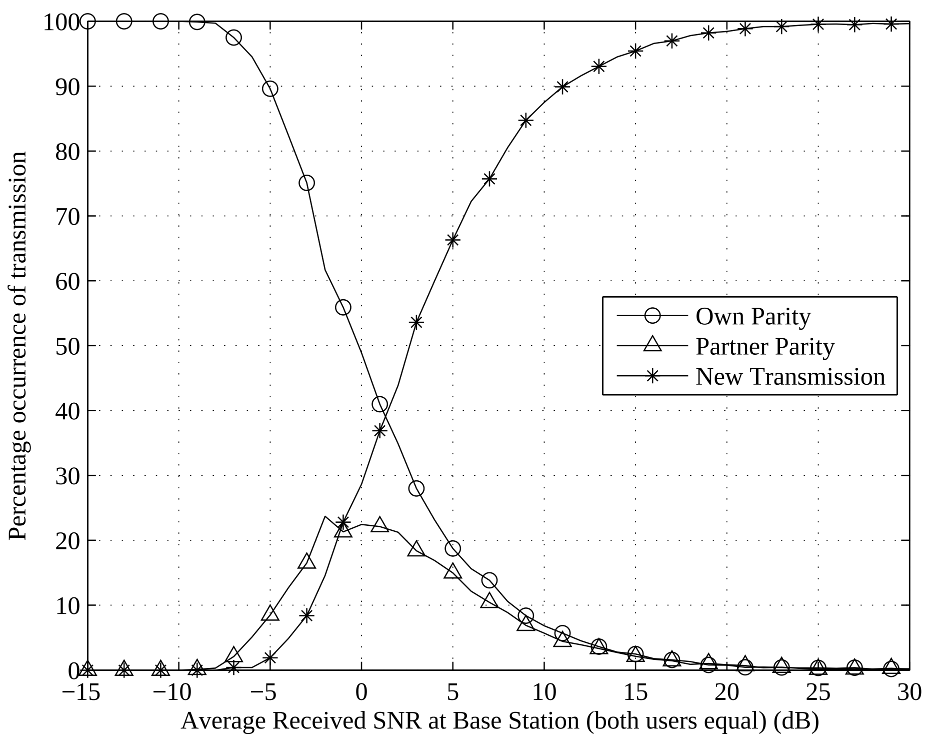


Figure 4.9: Protocol II – Percentage occurrence of each type of transmission (own parity, partner parity and new transmission) by user 1 for ACK/NACK based coded cooperative diversity with extended LDPC codes. Information bits = 512, 50% cooperation, both users equal SNR, very slow fading (block fading) channel and mutually independent inter-user channel with average SNR of 10 dB.

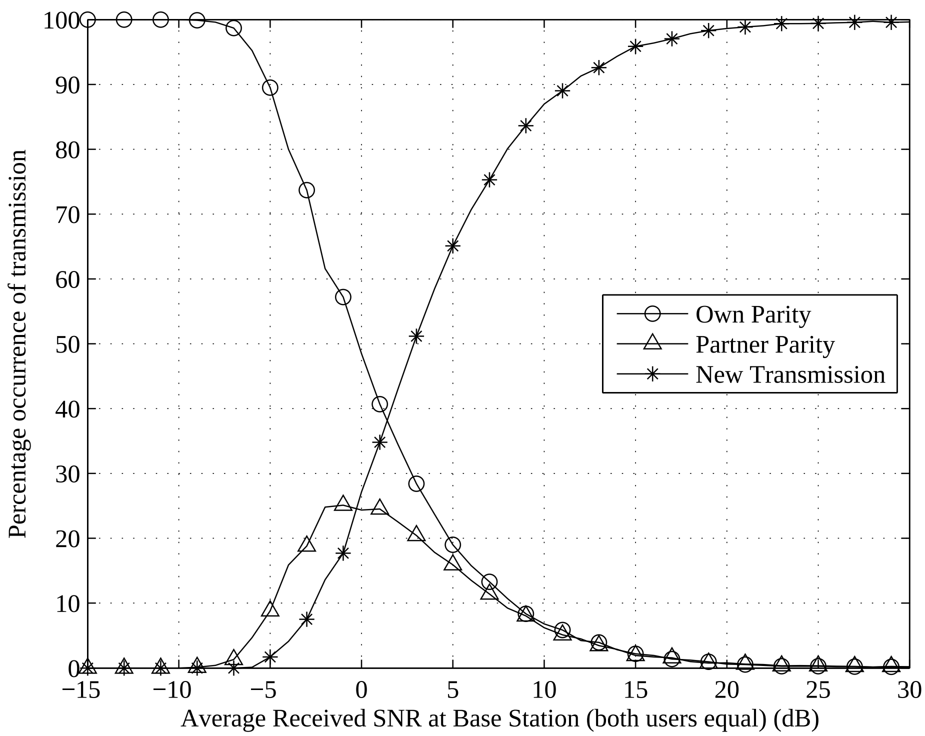


Figure 4.10: Protocol II – Percentage occurrence of each type of transmission (own parity, partner parity and new transmission) by user 1 for ACK/NACK based coded cooperative diversity with extended LDPC codes. Information bits = 512, 50% cooperation, both users equal SNR, very slow fading (block fading) channel, mutually independent inter-user channel and perfect inter-user channel condition.

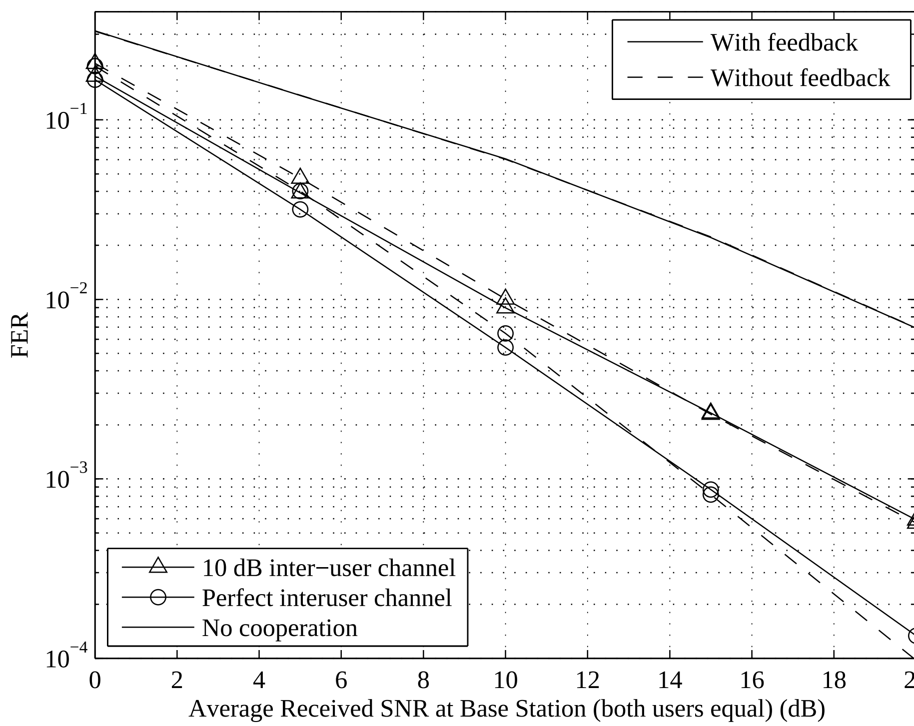


Figure 4.11: Protocol II – Effect on FER of Protocol II. A comparison of cooperative diversity with extended LDPC codes (FER) with and without feedback. Information bits = 512, 50% cooperation, both users equal SNR, very slow fading (block fading) channel.

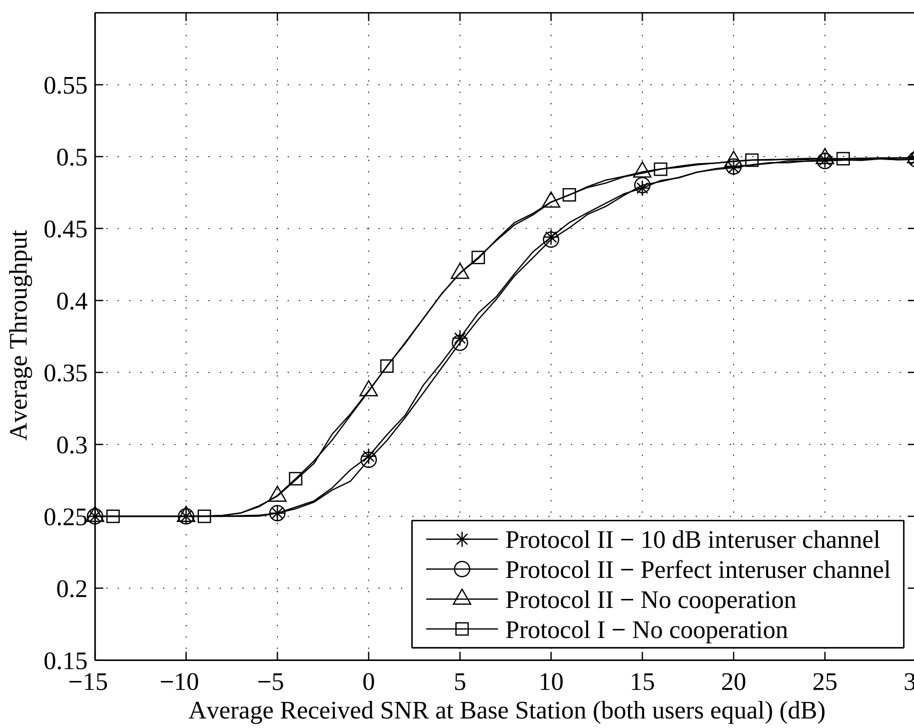


Figure 4.12: Protocol I & II – Average throughput for the ACK/NACK-based extended LDPC-coded cooperative diversity protocols. Information bits = 512, 50% cooperation, both users equal SNR, very slow fading (block fading) channel and mutually independent inter-user channel.

4.6 Conclusion

Two new ACK/NACK-based incremental relaying protocols for LDPC coded cooperative diversity has been proposed. The protocols outperforms the non-feedback based LDPC coded cooperative diversity schemes. The throughput of feedback-based protocols is almost double at higher SNR at the destination as compared to non-feedback-based protocol. The cooperation in terms of partner parity or partner codeword increases between users as the inter-user channel condition improves. Protocol I shows better performance in terms of throughput as compared to Protocol II. However, Protocol II shows slightly improved FER as compared to Protocol I.

CHAPTER 5

CONCLUSIONS AND FUTURE RECOMMENDATIONS

5.1 Conclusions

In this work, we successfully integrated LDPC codes in cooperative diversity. We proposed a modification to extended LDPC codes. The modified extended codes have been successfully integrated into cooperative diversity framework. The performance of punctured LDPC codes is better than modified extended codes in terms of error-rate at higher channel SNR at the destination. However, the decoding complexity of punctured LDPC codes is much higher than modified extended LDPC codes in cooperative diversity. Therefore, there is a trade-off between error-rate and encoding/decoding complexity for the punctured LDPC codes and modified extended LDPC codes in cooperative diversity.

We proposed two new ACK/NACK based cooperative diversity protocols. The proposed protocols have very high throughput efficiency as compared to non-acknowledgement based cooperative diversity protocol. The throughout efficiency for the feedback-based extended LDPC coded cooperative diversity protocols is almost double at higher SNR at the destination as compared to non-feedback-based coded cooperative diversity protocol.

5.2 Future Recommendations

This research work can be further extended with the following modifications to the existing work:

- In this work, regular LDPC codes are used. The same framework can be extended using extended LDPC codes design with irregular **H** matrix.
- There exist LDPC codes that are specifically designed for block fading channels. These codes can enhance the performance of coded cooperative diversity over block fading channels.
- The discussed coded cooperative diversity framework can be further investigated with space-time block codes which can lead into further improvement in achieving the capacity of the relay channels.
- This work was restricted to two users. However, the same framework can be generalized to any number of users.

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